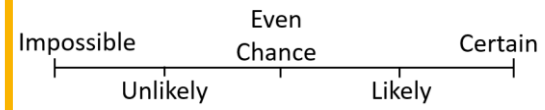
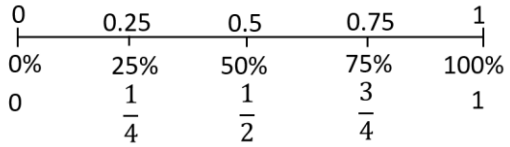
INTRODUCING PROBABILITY

Key Concept

Chance



Probability

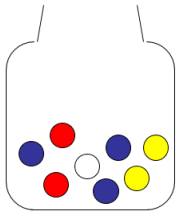


Probabilities can be written as:
- Fractions
- Decimals
- Percentages

Key Words

Probability: The chance of something happening as a numerical value.
Impossible: The outcome cannot happen.
Certain: The outcome will definitely happen.
Even chance: The are two different outcomes each with the same chance of happening.
Expectation: The amount of times you expect an outcome to happen based on probability.

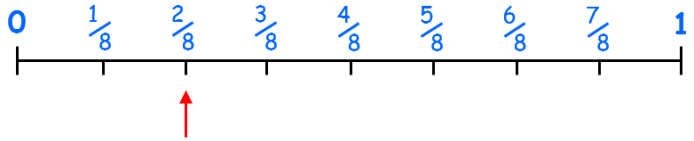
Examples



1) What is the probability that a bead chosen will be **yellow**.
Show the answer on a number line.

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$P(Yellow) = \frac{2}{8} = \frac{1}{4}$$



2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} of 40 = 10$$

sparx

Clip Numbers
M655,M941,
M938

Tip
Probabilities always add up to 1.

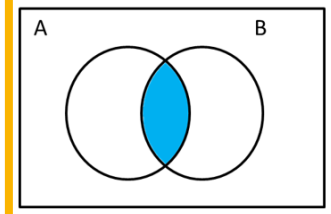
Formula
$Expectation = Probability \times no.\ of\ trials$

Questions
In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left.
Find: a) P(Red) b) P(Yellow) c) P(Red or purple) d) P(Green)

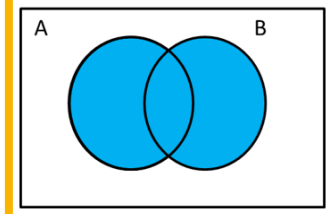
ANSWERS: 1) a) $\frac{12}{30} = \frac{2}{5}$ b) $\frac{9}{30} = \frac{3}{10}$ c) $\frac{15}{30} = \frac{1}{2}$ d) 0

FURTHER PROBABILITY

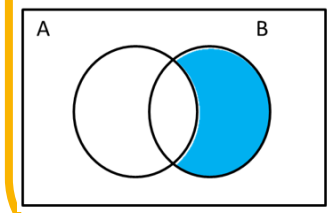
Key Concept



P(A ∩ B)



P(A U B)



P(A' ∩ B)

sparx

M718, M419
M829, M460

Key Words

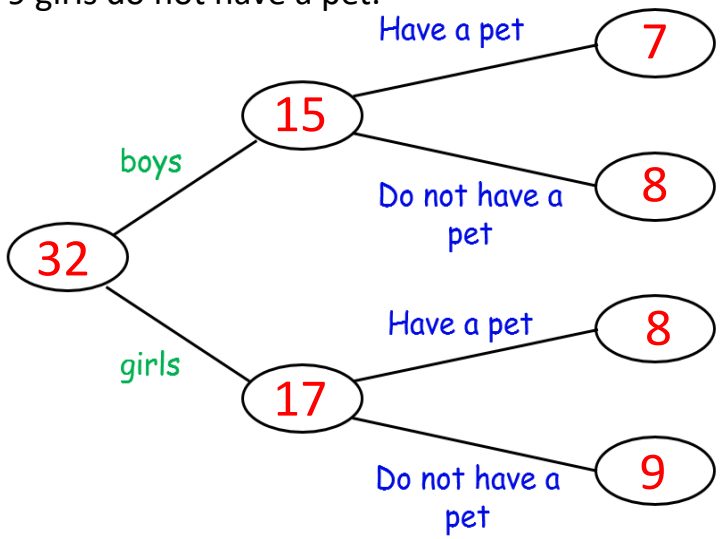
Probability: The chance of something happening as a numerical value.
Impossible: The outcome cannot happen.
Certain: The outcome will definitely happen.
Even chance: The are two different outcomes each with the same chance of happening.
Mutually Exclusive: Two events that cannot both occur at the same time.

Formula

$P(A \cap B) = P(A) \times P(B)$
$P(A \cup B) = P(A) + P(B)$
$or \quad (non\ ME) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Examples

In Hannah's class there are 32 students.
15 of these students are boys.
7 of the boys have a pet.
9 girls do not have a pet.



$$P(boy) = \frac{15}{32}$$

$$P(Girl\ with\ pet) = \frac{8}{32}$$

Questions

1) Draw a two-way table for the question above.
2) Find the probability that a pupil chosen is a boy with no pets.
3) A girl is chosen, what is the probability she has a pet?

ANSWERS: 2) $\frac{8}{32}$ 3) $\frac{8}{17}$

THEORETICAL PROBABILITY

Key Concepts

Probabilities can be described using **words** and **numerically**.

We can use **fractions, decimals or percentages** to represent a probability.

Theoretical probability is what should happen if all variables were fair.

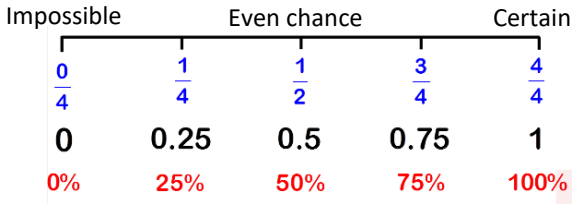
All probabilities must **add to 1**.

The probability of something **NOT** happening equals:

$1 - (probability\ of\ it\ happening)$

Examples

Probability scale:



There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3	5	2

1) What is the probability that a blue counter is chosen? $\frac{3}{19} = \frac{number\ of\ blue}{total\ number\ of\ counters}$

2) What is the probability that red is **not** chosen?

$$\frac{10}{19} = \frac{number\ of\ all\ other\ colours}{total\ number\ of\ counters}$$

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5
= 11

Probability of choosing black = $\frac{11}{100}$

sparx

U803 U408 U510

Key Words

Theoretical
Probability
Fraction
Decimal
Percentage
Certain
Impossible
Even chance

	1	2	3
Prob	5	4	9

1a) Calculate the probability of choosing a 2.
b) Calculate the probability of not choosing a 3.

	1	2	3
Prob	0.37	$2x$	x

2) Calculate the probability of choosing a 2 or a 3.

ANSWERS: 1a) $\frac{4}{18}$ b) $\frac{9}{18}$ 2) P(2) = 0.42 P(3) = 0.21

RELATIVE FREQUENCY

Key Concepts

Experimental probability differs to theoretical probability in that it is based upon the **outcomes from experiments**. It may not reflect the outcomes we expect.

Experimental probability is also known as the **relative frequency** of an event occurring.

Estimating the number of times an event will occur:

Probability × no. of trials

Examples

Colour	red	blue	white	black
Prob	x	0.2	0.3	x

A spinner is spun, it has four colours on it.
The relative frequencies of each colour are recorded.
The relative frequency of red and black are the same.

a) What is the relative frequency of red?

$$1 - (0.2 + 0.3) = 0.5$$
$$x = \frac{0.5}{2} = 0.25$$

b) If the spinner is spun 300 times, how many times do you expect it to land on white?

$$0.3 \times 300 = 90$$

sparx

U166 U580

Key Words

Experimental
Relative frequency
Fraction
Decimal
Probability
Estimate

Number	1	2	3	4
Prob	x	0.46	0.28	x

A spinner is spun which has 1,2,3,4 on it. The probability that a 1 and a 4 are spun are equal.

a) What is the probability that a 4 is landed on?

b) If the spinner is spun 500 times how many times do we expect it to land on a 2?

ANSWERS: a) 0.13 b) 230

PROBABILITY TREE DIAGRAMS

Key Concepts

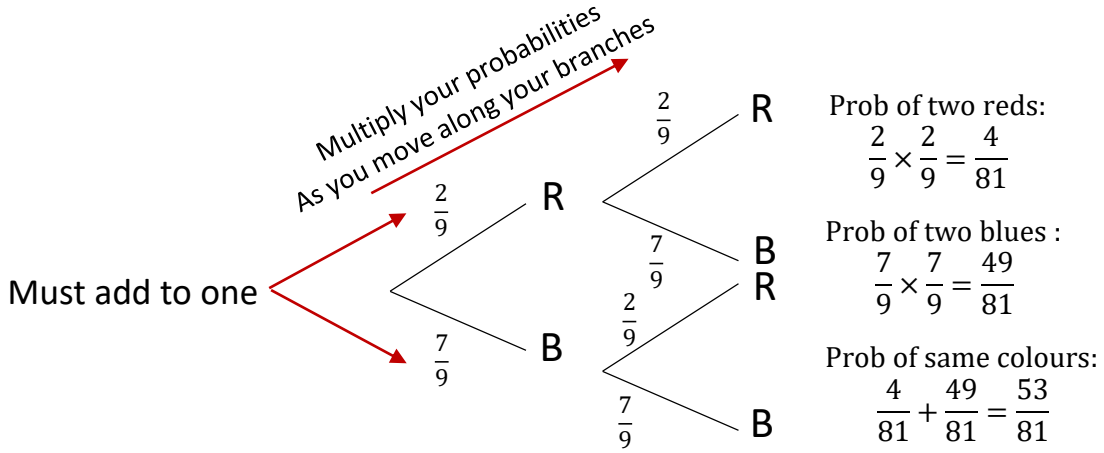
Independent events are events which do not affect one another.

Dependent events affect one another's probabilities. This is also known as **conditional probability**.

We **multiply** two probabilities when one event follows another.

Examples

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:
$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$

Prob of two blues :
$\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$

Prob of same colours:
$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$

sparx

U558 U729 U280 U296

Key Words
Independent
Dependant
Conditional
Probability
Fraction
Multiply

There are blue and green pens in a drawer.
There are 4 blues and 7 greens.
A pen is chosen and then **replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

ANSWERS: 56/121

PLOTTING AND INTERPRETTING GRAPHS

Key Concept

Substitution – This is where you replace a number with a letter

If a = 5 and b = 2

a + b =	5 + 2 = 7
a – b =	5 – 2 = 3
3a =	3 × 5 = 15
ab =	5 × 2 = 10
a^2 =	5^2 = 25

sparx

M932, M544,M888

Key Words

Intercept: Where two graphs cross.
Gradient: This describes the steepness of the line.
y-intercept: Where the graph crosses the y-axis.
Linear: A linear graph is a straight line.
Quadratic: A quadratic graph is curved, u or n shape.

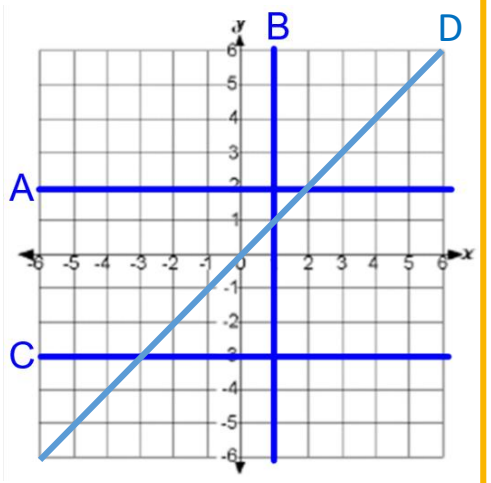
Tip

Parallel lines have the same gradient.

Formula

$$Gradient = \frac{difference\ in\ y's}{difference\ in\ x's}$$

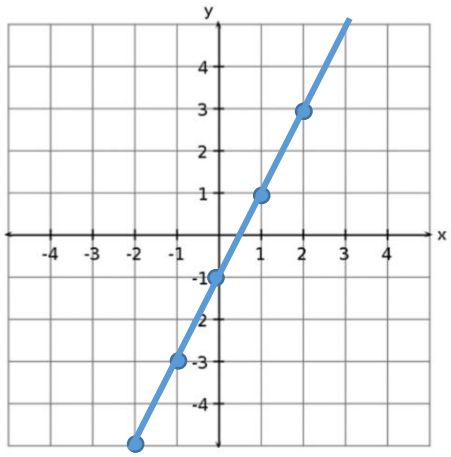
Examples



A: y = 2 B: x = 1
C: y = -3 D: y = x

Draw the graph of y = 2x - 1

X	- 2	- 1	0	1	2
Y	-5	-3	-1	1	3



Notice this graph has a gradient of 2 and a y-intercept of -1.

Questions

1) What are the gradient and y-intercept of:

a) y = 4x – 3 b) y = 4 + 6x c) y = - 5x – 3

2) Draw the graph of y = 3x – 2 for x values from -3 to 3 using a table

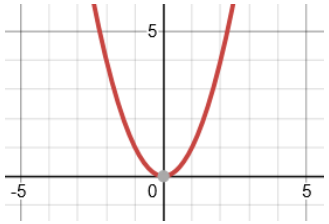
ANSWERS: 1) a) m = 4, c = -3 b) m = 6, c = 4 c) m = -5, c = -3

QUADRATIC GRAPHS

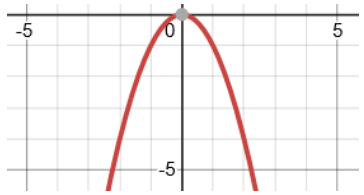
Key Concepts

A quadratic graph will always be in the shape of a parabola.

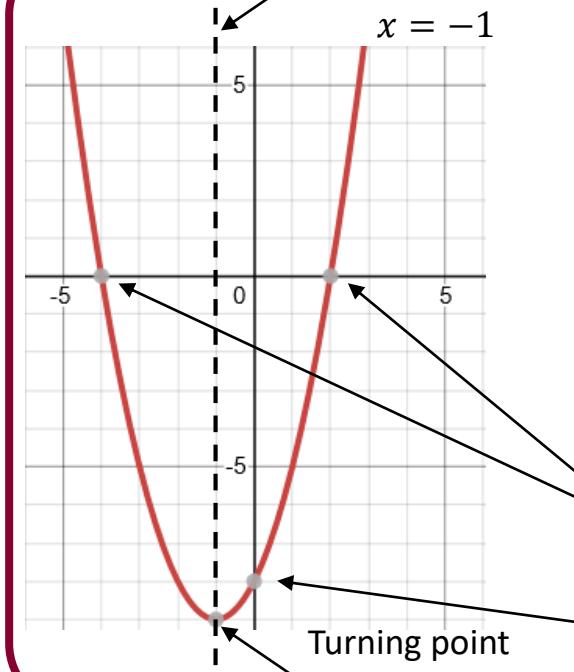
$y = x^2$



$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.



Examples

$$y = x^2 + 2x - 8$$

A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the x axis.

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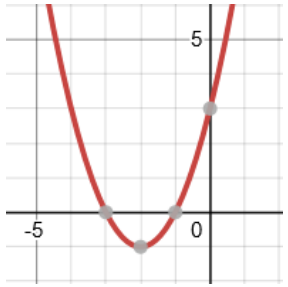
Key Words
Quadratic
Roots
Intercept
Turning point
Line of symmetry



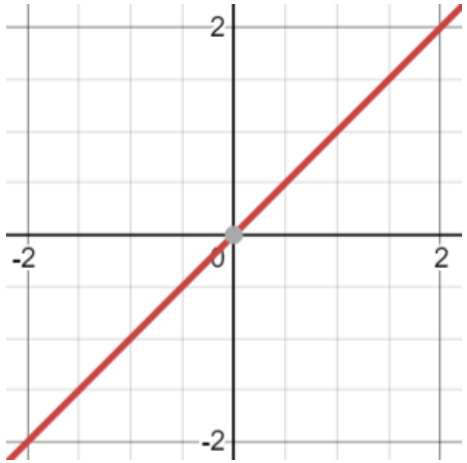
Identify from the graph of $y = x^2 + 4x + 3$:

1) The line of symmetry
2) The turning point
3) The y intercept
4) The two roots of the equation

ANSWERS 1) $x = -2$ 2) $(-2, -1)$ 3) 3 4) $x = -1$ and $x = -3$

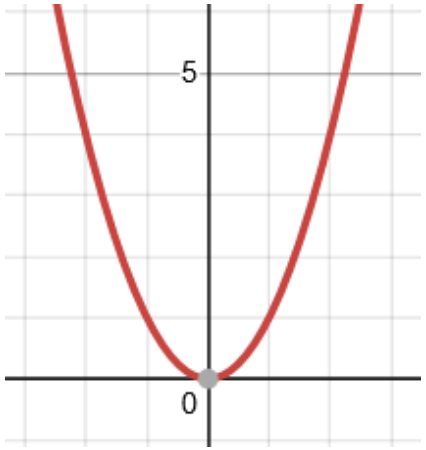
TYPES OF GRAPH

Examples



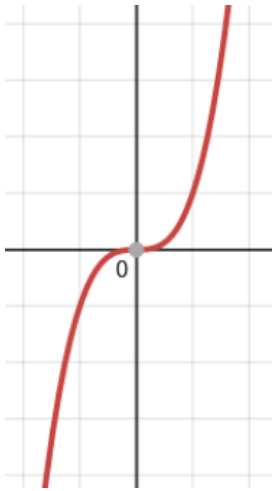
Linear graphs

$y = x$



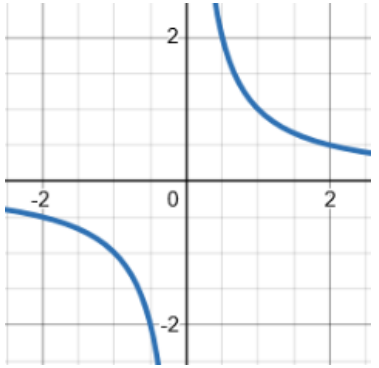
Quadratic graphs

$y = x^2$



Cubic graphs

$y = x^3$



Reciprocal graphs

$y = \frac{1}{x}$

sparx

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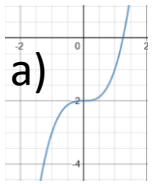
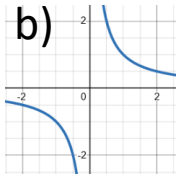
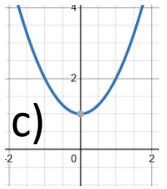
Key Words

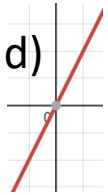
Quadratic
Cubic
Reciprocal
Linear
Graph

Match the graph with the correct equation:

a) 

b) 

c) 

d) 

1) $y = 2x$
2) $y = \frac{1}{x}$
3) $y = x^3 - 2$
4) $y = x^2 + 1$

ANSWERS: 1d 2b 3a 4c

SIMULTANEOUS EQUATIONS

Key Concepts

Simultaneous equations are when **more than one equation** are given, which involve **more than one variable**. The variables have the **same value** in each equation.

Example

We need to make the y coefficients the same

$$3x + 2y = 18$$
$$3x - y = 9 \quad \times 2$$
$$3x + 2y = 18$$
$$6x - 2y = 18 \quad +$$

SSS – Same Sign Subtract
DSA – Different Sign Add

$$9x = 36$$
$$x = 4$$

Substitute $x = 4$ into an original equation:

$$3x + 2y = 18$$
$$(3 \times 4) + 2y = 18$$
$$12 + 2y = 18$$
$$2y = 6$$
$$y = 3$$

Check in the other equation:

$$(3 \times 4) - 3 = 9$$
$$12 - 3 = 9$$

This is true therefore $x = 4$ *and* $y = 3$

sparx

U760, U757, U137

Key Words

Simultaneous
Substitution
Elimination
Linear

Solve each set of simultaneous equations:

1) $3x + 2y = 36$
$5x + 4y = 64$

2) $3x + 2y = 4$
$4x + 5y = 17$

ANSWERS: 1) $x = 8$ *and* $y = 6$ 2) $x = -2$ *and* $y = 5$

INEQUALITIES

Key Concepts

Inequalities show the **range** of numbers that satisfy a rule.

$x < 2$ means x is less than 2
$x \leq 2$ means x is less than or equal to 2
$x > 2$ means x is greater than 2
$x \geq 2$ means x is greater than or equal to 2

On a **number line** we use circles to highlight the key values:

○ is used for less/greater than
● is used for less/greater than or equal to

Examples

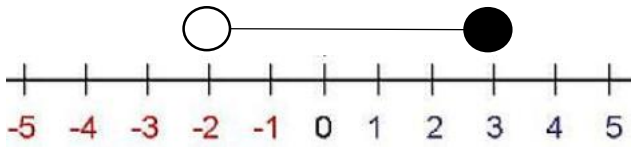
a) State the values of n that satisfy:

$$-2 < n \leq 3$$

Cannot be equal to 2 Can be equal to 3

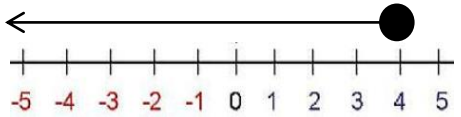
-1, 0, 1, 2, 3

b) Show this inequality on a number line:



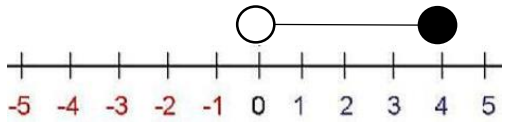
Solve this inequality and represent your answer on a **number line:**

$5x - 6 \leq 14$
+6 +6
$5x \leq 20$
÷5 ÷5
$x \leq 4$



Solve this inequality and represent your answer on a **number line:**

$4 < 3x + 1 \leq 13$
-1 -1
$3 < 3x \leq 12$
÷3 ÷3
$1 < x \leq 4$



sparx
M384
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Key Words
Inequality
Greater than
Less than
Represent
Number line

1) State the values of n that satisfy: $-3 \leq n < 2$
2) Solve $4x - 2 \leq 6$ and represent your answer on a number line
3) Solve $5 < 2x + 3 \leq 9$ and represent your answer on a number line

ANSWERS: 1) -3, -2, -1, 0, 1 2) $x \leq 2$ 3) $1 < x \leq 3$