FRACTIONS & PERCENTAGES AS OPERATORS

Key Concept

Multipliers

Find 15%	$\times 0.15$
Increase by 15%	$\times 1.15$
Decrease by 15%	$\times 0.85$

For **reverse percentage** problems you can divide by the multiplier to find the original amount.

Key Words

Percentage: Is a proportion that shows a number as parts per hundred.
Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).
Multiplier: A quantity by which a given number is to be multiplied.

Examples

Non-Calculator

$$\frac{3}{4}\ of\ 32 = 32 \div 4 \times 3 = 24$$

$16\%\ of\ 240$

$10\% = 24$
$5\% = 12$
$1\% = 2.4$

$= 24 + 12 + 2.4$
$= 38.4$

Calculator

Find **32%** of 54.60 = **0.32** × 54.60 = 17.472

Increase 45 by **12%** = 45 × **1.12** = 50.4

sparx

M157,U475
M958,M264,U88
M437

Tip
There is a % function on your calculator.

To find 25% of 14 on a calculator:
2, 5, SHIFT, (, ×, 1, 4, =

Questions

1) Find these fractions of amounts:
a) $\frac{1}{3}\ of\ 15$ a) $\frac{1}{5}\ of\ 65$ a) $\frac{2}{7}\ of\ 14$ a) $\frac{4}{9}\ of\ 45$
2) a) 35% of 140 b) 21% of 360 c) Increase 60 by 15%

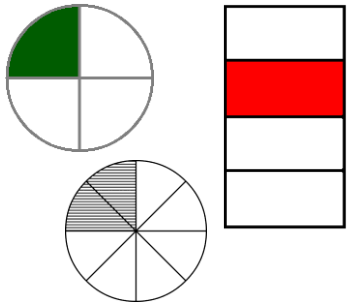
ANSWERS: 1) a) 5 b) 13 c) 4 d) 20 2) a) 49 b) 75.6 c) 69

UNDERSTANDING FRACTIONS

Key Concept

$$\frac{1}{4}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16}$$



= 0.25

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).
Equivalence: Two fractions are equivalent if one is a multiple of the other.
Simplify: Cancel a fraction down to give the smallest numbers possible.

Examples

Simplify $\frac{3}{24}$ a fraction

$\frac{3}{24}$ $\xrightarrow{\div 3}$ $\frac{1}{8}$ $\xrightarrow{\div 3}$ $\frac{1}{8}$

Write 0.34 as

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	3	4	

$$= \frac{34}{100} = \frac{17}{50}$$

Write $\frac{6}{7}$ as a decimal

$$\frac{6}{7} \rightarrow 7\overline{)6.00000}$$

0 . 8 5 7 1 4
6 4 5 1 3

What is $\frac{3}{5}$ *of* 45

$45 \div 5 \times 3 = 27$

sparx

M158, M410, M601, M835, M157, M197

Tip

- A larger denominator does not mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

Questions

1) Simplify a) $\frac{42}{96}$ b) $\frac{64}{120}$ 2) Write as a decimal a) $\frac{2}{7}$ b) $\frac{3}{8}$

3) Write as a fraction a) 0.48 b) 0.166 c) 0.308

4) a) $\frac{3}{5}$ *of* 35 b) $\frac{2}{9}$ *of* 45 c) $\frac{5}{11}$ *of* 121

ANSWERS : 1) a) $\frac{7}{16}$ b) $\frac{8}{15}$ 2) a) 0.2857... b) 0.375

3) a) $\frac{12}{25}$ b) $\frac{83}{500}$ c) $\frac{77}{250}$

4) a) 21 b) 10 c) 55

FOUR OPERATIONS WITH FRACTIONS

Key Concept

Mixed numbers

These are made up of a whole number and a fraction.

$$4\frac{3}{5}$$

$$= \frac{4 \times 5 + 3}{5}$$

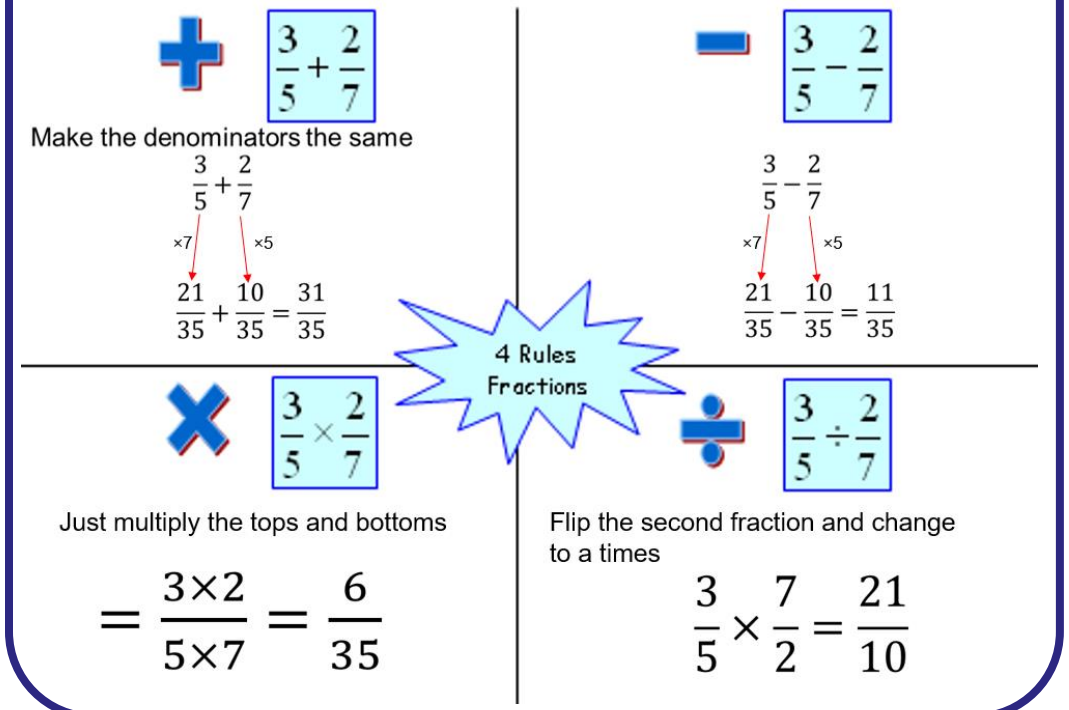
$$= \frac{23}{5}$$

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Add Sum Total All together Plus In all	Multiply Product Times Twice Total Multiplied by
Subtract Remain Difference Less than Fewer How many more Minus	Divide Quotient Goes into Split Equally Each

Examples



sparx

M671, M939, M601, M835, M931, M157, M197, M110

Tip

- A larger denominator **does not** mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

Questions

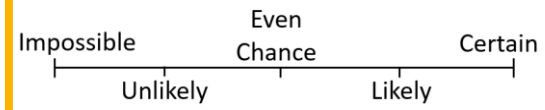
1) $\frac{3}{5}+\frac{4}{15}$ 2) $\frac{2}{7}+\frac{5}{8}$ 3) $\frac{7}{9}-\frac{2}{5}$ 4) $\frac{3}{7}\times\frac{4}{9}$ 5) $\frac{3}{11}\div\frac{14}{22}$

ANSWERS: 1) $\frac{13}{15}$ 2) $\frac{51}{56}$ 3) $\frac{17}{45}$ 4) $\frac{4}{21}$ 5) $\frac{3}{7}$

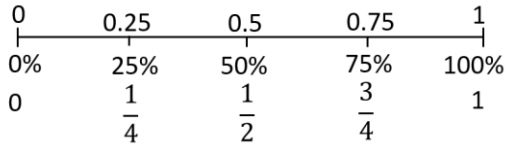
INTRODUCING PROBABILITY

Key Concept

Chance



Probability

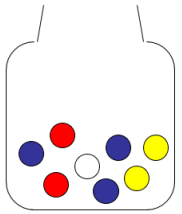


Probabilities can be written as:
- Fractions
- Decimals
- Percentages

Key Words

Probability: The chance of something happening as a numerical value.
Impossible: The outcome cannot happen.
Certain: The outcome will definitely happen.
Even chance: The are two different outcomes each with the same chance of happening.
Expectation: The amount of times you expect an outcome to happen based on probability.

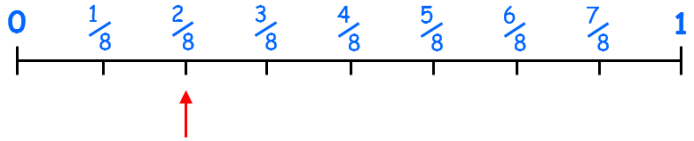
Examples



1) What is the probability that a bead chosen will be **yellow**.
Show the answer on a number line.

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$P(Yellow) = \frac{2}{8} = \frac{1}{4}$$



2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} of 40 = 10$$

sparx

Clip Numbers
M655,M941,
M938

Tip
Probabilities always add up to 1.

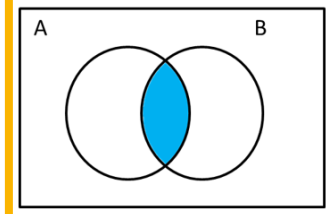
Formula
$Expectation = Probability \times no.\ of\ trials$

Questions
In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left.
Find: a) P(Red) b) P(Yellow) c) P(Red or purple) d) P(Green)

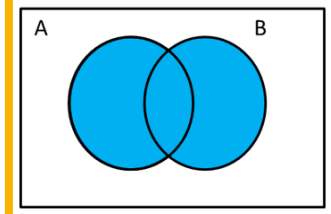
ANSWERS: 1) a) $\frac{12}{30} = \frac{2}{5}$ b) $\frac{9}{30} = \frac{3}{10}$ c) $\frac{15}{30} = \frac{1}{2}$ d) 0

FURTHER PROBABILITY

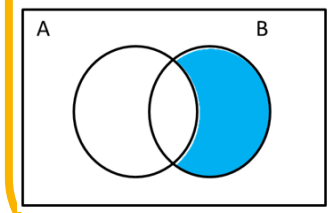
Key Concept



P(A ∩ B)



P(A U B)



P(A' ∩ B)

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M718, M419
M829, M460

Key Words

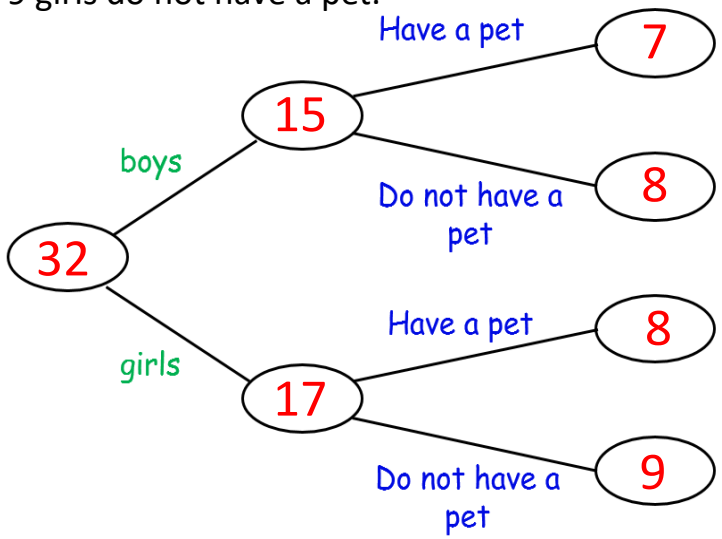
Probability: The chance of something happening as a numerical value.
Impossible: The outcome cannot happen.
Certain: The outcome will definitely happen.
Even chance: The are two different outcomes each with the same chance of happening.
Mutually Exclusive: Two events that cannot both occur at the same time.

Formula

$P(A \cap B) = P(A) \times P(B)$
$P(A \cup B) = P(A) + P(B)$
$or \quad (non\ ME) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Examples

In Hannah's class there are 32 students.
15 of these students are boys.
7 of the boys have a pet.
9 girls do not have a pet.



$$P(boy) = \frac{15}{32}$$

$$P(Girl\ with\ pet) = \frac{8}{32}$$

Questions

1) Draw a two-way table for the question above.
2) Find the probability that a pupil chosen is a boy with no pets.
3) A girl is chosen, what is the probability she has a pet?

ANSWERS: 2) $\frac{8}{32}$ 3) $\frac{8}{17}$

FACTORS, MULTIPLES AND PRIMES

Key Concept

Factors:
Find these in pairs
12
1, 12
2, 6
3, 4

Multiples:
Start with the number itself
7 – 7, 14, 21, 28, …

Key Words

Factor: The numbers which fit into a number exactly.
Multiple: The numbers in the times table.
Prime: Numbers which have only two factors which are 1 and itself.
Highest Common Factor: The highest factor which is common for both numbers.
Lowest Common Multiple: The smallest multiple which is common to both numbers.

Examples

Lowest Common Multiple (LCM)

Q - Find the LCM of 6 and 7:
6 – 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, …
7 – 7, 14, 21, 28, 35, 42, 49, 56, …
LCM = 42

Highest Common Factor (HCF)

Q – Find the HCF of 18 and 24
18 – 1, 2, 3, 6, 9, 18
24 – 1, 2, 3, 4, 6, 8, 12, 24
HCF = 6

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M462
M823
M322

Tip

There is only one even prime number which is the number 2. This can be used to help solve lots of problems.

Questions

1) List the first 5 multiples of: a) 7 b) 12 c) 50
2) List the factors of: a) 12 b) 15 c) 16
3) a) Find the LCM of 5 and 7 b) Find the HCF of 20 and 16

ANSWERS: 1) a) 7, 14, 21, 28, 35 b) 12, 24, 36, 48, 60 c) 50, 100, 150, 200, 250
2) a) 1, 2, 3, 4, 6, 12 b) 1, 3, 5, 15 c) 1, 2, 4, 8, 16 3) a) 35 b) 4