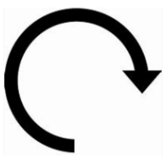
REFLECTION AND ROTATION

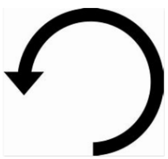
Key Concepts

A **reflection** creates a mirror image of a shape on a coordinate graph. The mirror line is given by an equation eg. $y = 2, x = 2, y = x$. The shape does not change in size.

A **rotation** turns a shape on a coordinate grid from a given point. The shape does not change size but does change orientation.



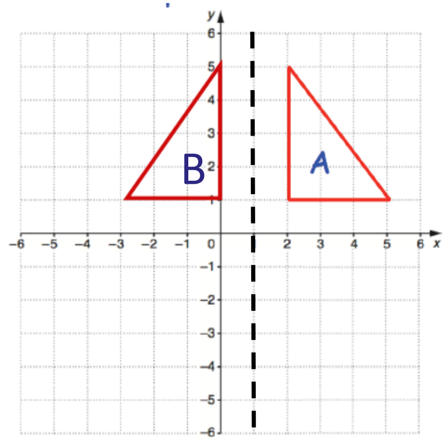
Clockwise



Anticlockwise

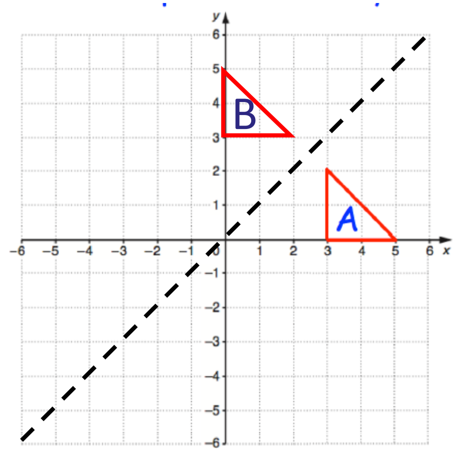
Examples

Reflect shape A in the line $x = 1$. Label it B.



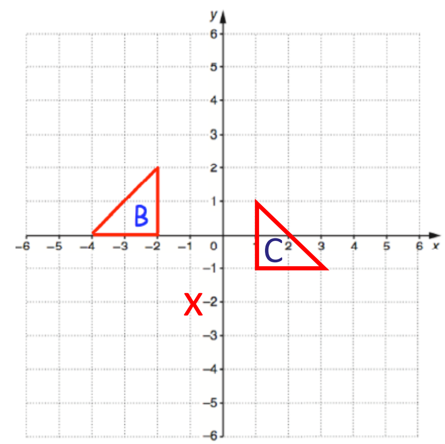
$x = 1$

Reflect shape A in the line $y = x$. Label it B.



$y = x$

Rotate shape B from the point (-1, -2)



sparx

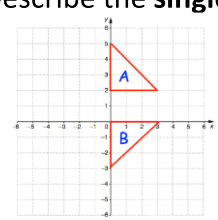
U799

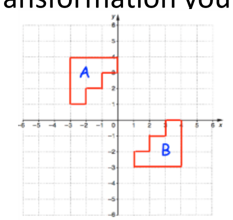
U696

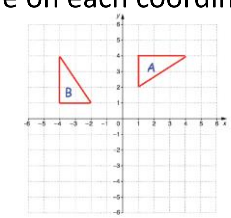
Key Words

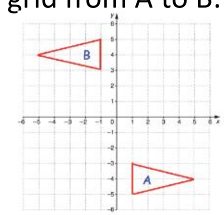
Rotate
Clockwise
Anticlockwise
Centre
Degrees
Reflect
Mirror image

Describe the **single** transformation you see on each coordinate grid from A to B:









ANSWERS: a) reflection, $y = 1$ b) reflection $y = x$ c) rotation, centre (0,0), 90° anticlockwise
d) rotation, centre (0,0), 180°

TRANSLATION AND ENLARGEMENT

Key Concepts

A **translation** moves a shape on a coordinate grid. Vectors are used to instruct the movement:

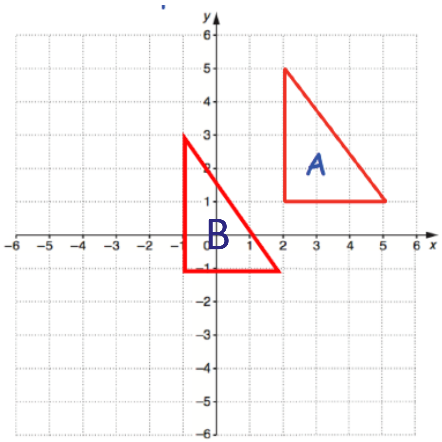
$\begin{pmatrix} x \\ y \end{pmatrix}$

- x: Positive-Right, Negative - Left
- y: Positive-Up, Negative - Down

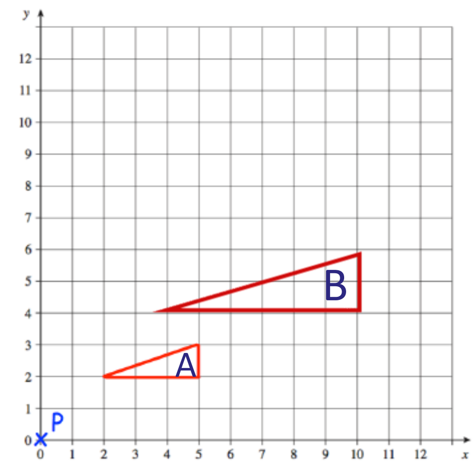
An **enlargement** changes the size of an image using a scale factor from a given point.

Examples

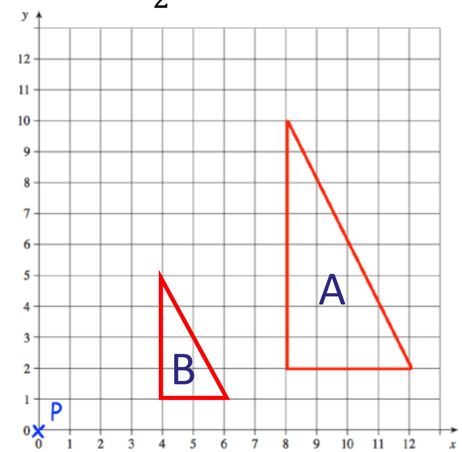
Translate shape A by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$. Label it B



Enlarge shape A by scale factor 2 from point P.



Enlarge shape A by scale factor $\frac{1}{2}$ from point P.



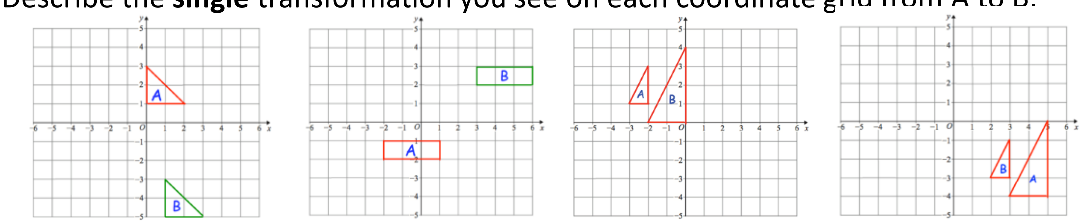
sparx

U196
U519
U134

Key Words

Translation
Enlargement
Scale factor
Centre
Positive
Negative

Describe the **single** transformation you see on each coordinate grid from A to B:

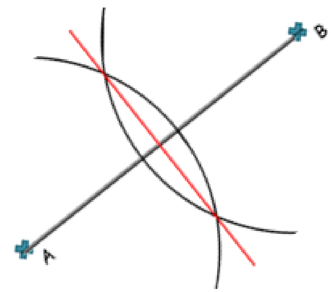


ANSWERS: a) translation $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ b) translation $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ c) enlarge, centre (-4,2) scale factor 2
d) enlarge, centre (1,-2) scale factor $\frac{1}{2}$

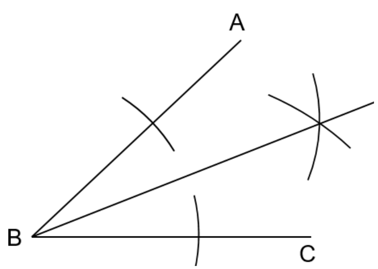
CONSTRUCTIONS AND LOCI

Key Concepts

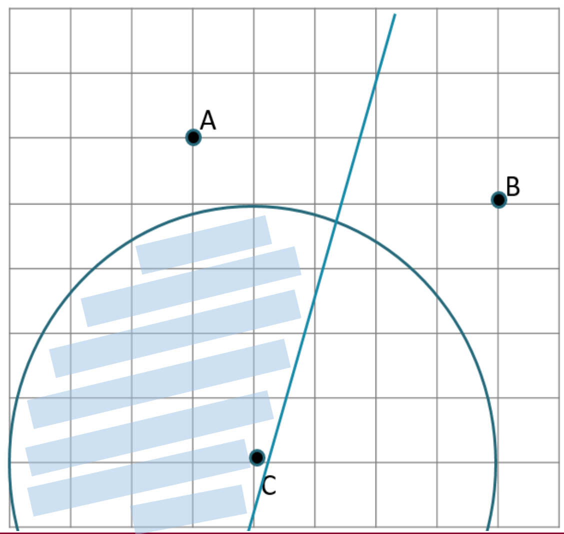
Line bisector



Angle bisector



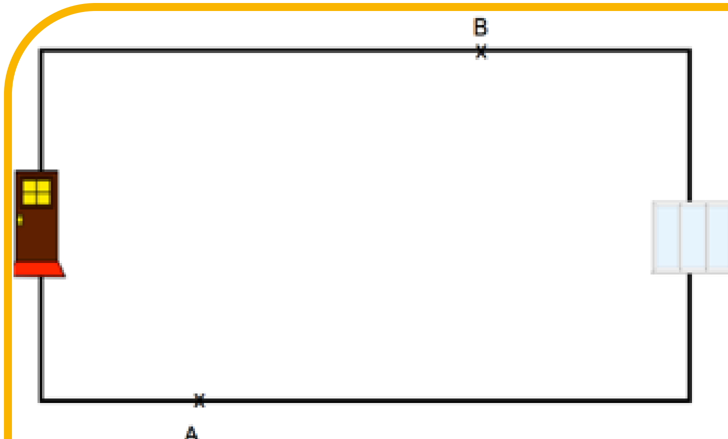
Examples



Shade the region that is:
- closer to A than B
- less than 4 cm from C

Line bisector of A and B

Circle with radius 4cm



1cm = 1m

There are two burglar alarm sensors, one at A and one at B.

The range of each sensor is 4m.

The alarm is switched on.

Is it possible to walk from the front door to the patio door without setting off the alarm?

Key Words

Bisect
Radius
Region
Shade

sparx

U187, U245, U787, U979, U820

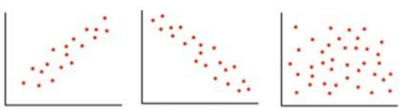
PIE CHARTS AND SCATTER-GRAPHS

Key Concepts

Pie charts use angles to represent proportionally the quantity of each group involved.

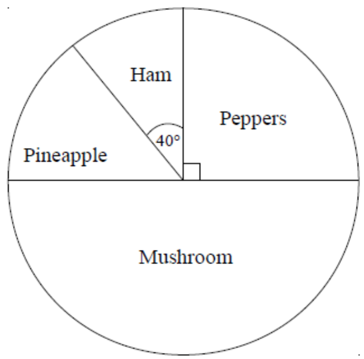
Pie charts can only be compared to one another when populations are given.

Scatter-graphs show the relationship between two variables. This relationship is called the **correlation**.



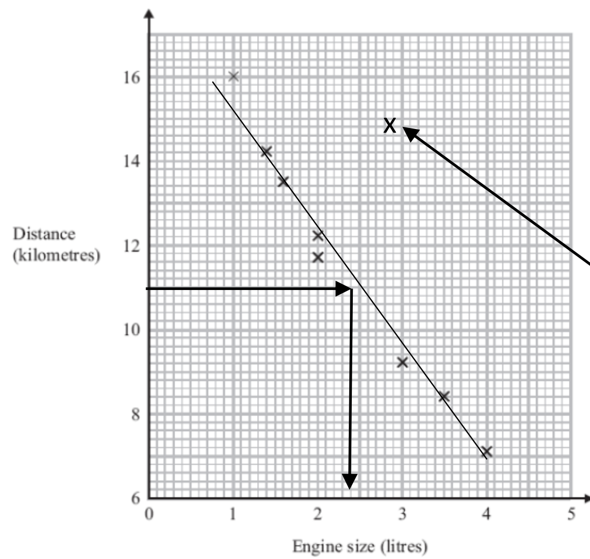
Positive **Negative** **No Correlation**

Examples



Topping	Frequency	Angle of Sector
Peppers	18	90°
Mushroom	36	180°
Pineapple	10	50°
Ham	8	40°
	72	360°

$$\frac{360}{72} = 5 \rightarrow \times 5$$



A scatter-graph is drawn to show the relationship between the engine size of a car and how far it can travel.

It shows negative correlation.

This is an outlier.

We draw a line of best fit through the middle of the data points to read from to estimate readings. For example, estimating the engine size of a car that can travel 11km would be 2.5 litres.

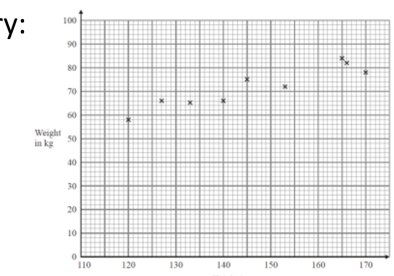
sparx

U508 U172 U854
U199 U277 U128

Key Words
Pie chart
Scatter-graph
Correlation
Outlier
Variable

1) Calculate the angle for each category:

Region	Frequency
Southern England	9
London	23
Midlands	16
Northern England	12
Total	60



2a) What type of correlation is shown?
b) Using a line of best fit estimate the weight when the height is 135cm.

ANSWERS: 1) 54, 138, 96, 72 2) a) positive b) 64kg-66kg

THEORETICAL PROBABILITY

Key Concepts

Probabilities can be described using **words** and **numerically**.

We can use **fractions, decimals or percentages** to represent a probability.

Theoretical probability is what should happen if all variables were fair.

All probabilities must **add to 1**.

The probability of something **NOT** happening equals:

$$1 - (probability\ of\ it\ happening)$$

Examples

Probability scale:

Impossible		Even chance		Certain
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3	5	2

1) What is the probability that a blue counter is chosen? $\frac{3}{19} = \frac{number\ of\ blue}{total\ number\ of\ counters}$

2) What is the probability that red is **not** chosen?

$$\frac{10}{19} = \frac{number\ of\ all\ other\ colours}{total\ number\ of\ counters}$$

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5
= 11

Probability of choosing black = $\frac{11}{100}$

U803 U408 U510

Key Words

Theoretical
Probability
Fraction
Decimal
Percentage
Certain
Impossible
Even chance

	1	2	3
Prob	5	4	9

1a) Calculate the probability of choosing a 2.
b) Calculate the probability of not choosing a 3.

	1	2	3
Prob	0.37	$2x$	x

2) Calculate the probability of choosing a 2 or a 3.

ANSWERS: 1a) $\frac{4}{18}$ b) $\frac{9}{18}$ 2) P(2) = 0.42 P(3) = 0.21

RELATIVE FREQUENCY

Key Concepts

Experimental probability differs to theoretical probability in that it is based upon the **outcomes from experiments**. It may not reflect the outcomes we expect.

Experimental probability is also known as the **relative frequency** of an event occurring.

Estimating the number of times an event will occur:

Probability × no. of trials

Examples

Colour	red	blue	white	black
Prob	x	0.2	0.3	x

A spinner is spun, it has four colours on it.
The relative frequencies of each colour are recorded.
The relative frequency of red and black are the same.

a) What is the relative frequency of red?

$$1-(0.2+0.3)=0.5$$

$$x=\frac{0.5}{2}=0.25$$

b) If the spinner is spun 300 times, how many times do you expect it to land on white?

$$0.3\times 300=90$$

sparx

U166 U580

Key Words

Experimental
Relative frequency
Fraction
Decimal
Probability
Estimate

Number	1	2	3	4
Prob	x	0.46	0.28	x

A spinner is spun which has 1,2,3,4 on it. The probability that a 1 and a 4 are spun are equal.

a) What is the probability that a 4 is landed on?

b) If the spinner is spun 500 times how many times do we expect it to land on a 2?

ANSWERS: a) 0.13 b) 230

ALGEBRAIC PROOF

Examples

Prove:

$$(\boldsymbol{n}+\mathbf{4})^2-(\boldsymbol{n}+\mathbf{2})^2$$

is always a multiple of 4 for all positive integers of n.

$$(\boldsymbol{n}+\mathbf{4})^2-(\boldsymbol{n}+\mathbf{2})^2$$

Expand Expand

$$(\boldsymbol{n}^2+\mathbf{8}\boldsymbol{n}+\mathbf{16})-(\boldsymbol{n}^2+\mathbf{4}\boldsymbol{n}+\mathbf{4})$$

Simplify Simplify

$$\mathbf{4}\boldsymbol{n}+\mathbf{12}$$

Factorise Factorise

$$\mathbf{4}(\boldsymbol{n}+\mathbf{3})$$

Because 4 is a factor of all terms in this expression, then the original expression must always be a multiple of 4.

Prove that the sum of any three consecutive even numbers is always a multiple of 6:

Term 1: $2n$
Term 2: $2n + 2$
Term 3: $2n + 4$

$$\mathbf{2}\boldsymbol{n}+\mathbf{2}\boldsymbol{n}+\mathbf{2}+\mathbf{2}\boldsymbol{n}+\mathbf{4}$$

Simplify Simplify

$$\mathbf{6}\boldsymbol{n}+\mathbf{6}$$

Factorise Factorise

$$\mathbf{6}(\boldsymbol{n}+\mathbf{1})$$

6 is a factor of all terms therefore the original expression must always be a multiple of 6.

Prove that the product of two odd numbers is always odd:

Term 1: $2n$+1
Term 2: $2n + 3$

$$(\mathbf{2}\boldsymbol{n}+\mathbf{1})(\mathbf{2}\boldsymbol{n}+\mathbf{3})$$

Expand Expand

$$\mathbf{4}\boldsymbol{n}^2+\mathbf{8}\boldsymbol{n}+\mathbf{3}$$

Factorise Factorise

$$\mathbf{4}\boldsymbol{n}(\boldsymbol{n}+\mathbf{2})+\mathbf{3}$$

This term is even as any multiple of 4 is even.

This term is odd as 3 is an odd number.

Even + Odd = Odd number

sparx

U582

Key Words

Term
Odd
Even
Consecutive
Sum
Product

1) Prove $(n+10)^2-(n+2)^2$ is always a multiple of 16.
2) Prove the sum of two consecutive odd numbers is even.
3) Prove the product of two even consecutive numbers is always a multiple of 4.

ANSWERS: 1) $16(n+6)$ 2) $2(2n+2)$ 3) $4(n^2+n)$