DIRECT AND INVERSE PROPORTION

Key Concepts

Variables are **directly proportional** when the **ratio is constant** between the quantities.

Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing.**

Examples

Direct proportion:

Value of A	32	P	56	20	72
Value of B	20	30	35	R	45

Ratio constant: $20 \div 32 = \frac{5}{8}$

From A to B we will multiply by $\frac{5}{8}$.

From B to A we will divide by $\frac{5}{8}$.

$P = 30 \div \frac{5}{8} = 48$

$R = 20 \times \frac{5}{8} = 12.5$

Inverse proportion:

÷5

×2

Value of A	10	20	14	R	28
Value of B	14	P	10	70	5

÷2

×5

P = 7

R = 2

M478

M681

Key Words

Direct

Inverse

Proportion

Divide

Multiply

Constant

Complete each table:

1) Direct proportion

Value of A	5	P	22
Value of B	9	28.8	Q

2) Inverse proportion

Value of A	4	P	18
Value of B	9	3	Q

ANSWERS 1) P = 16, Q = 39.6 2) P = 12, Q = 2

RATIO AND DIRECT PROPORTION

Key Concepts

To calculate the **value** for a single item we can use the **unitary method**.

When working with best value in monetary terms we use:

$$Price\ per\ unit = \frac{price}{quantity}$$

In recipe terms we use:

$$Weight\ per\ unit = \frac{weight}{quantity}$$

If 20 apples weigh 600g. How much would 28 apples weigh?

600 ÷ 20 = 30g ⟶ weight of 1 apple

30 × 28 = **840g**

Box A has 8 fish fingers costing £1.40.
Box B has 20 fish fingers costing £ 3.40.
Which box is the better value?

$$A = \frac{£1.40}{8} = £0.175 \qquad B = \frac{£3.40}{20} = £0.17$$

Therefore Box B is better value as each fish finger costs less.

Examples

The recipe shows the ingredients needed to make 10 Flapjacks.
How much of each will be needed to make 25 flapjacks?

Ingredients for 10 Flapjacks

80 g rolled oats

60 g butter

30 *ml* golden syrup

36 g light brown sugar

Method 1: Unitary

80 ÷ 10 = 8
8 × 25 = **200g**

30 ÷ 10 = 3
3 × 25 = **75g**

60 ÷ 10 = 6
6 × 25 = **150g**

36 ÷ 10 = 3.6
3.6 × 25 = **90g**

Method 2: 5 flapjacks

80 ÷ 2 = 40
40 × 5 = **200g**

30 ÷ 2 = 15
15 × 5 = **75g**

60 ÷ 2 = 30
30 × 5 = **150g**

36 ÷ 2 = 18
18 × 5 = **90g**

sparx

M525

M801

Key Words

Unitary
Best Value
Proportion
Quantity

Ingredients
to make 16 gingerbread men

180 g	flour
40 g	ginger
110 g	butter
30 g	sugar

1) How much will we need to make 24 gingerbread men?

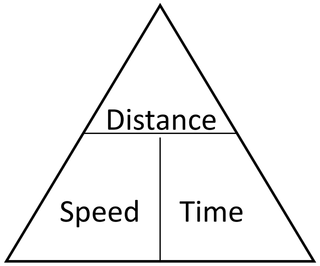
2) Packet A has 10 toilet rolls costing £3.50.
Packet B has 12 toilet rolls costing £3.60.
Which is better value for money?

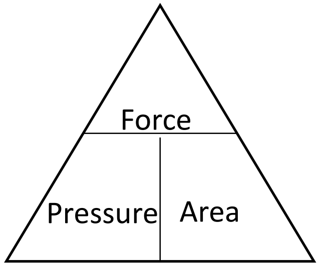
3) If 15 oranges weigh 300g. What will 25 oranges weigh?

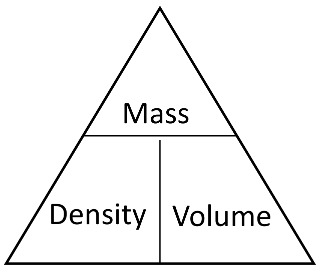
ANSWERS 1) 270g flour, 60g ginger, 165g butter, 45g sugar 2) Packet B 30p per roll 3) 500g

COMPOUND MEASURES

Key Concepts





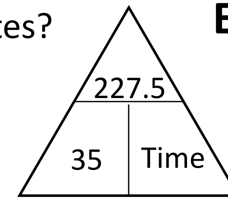


Examples

A car is travelling at a speed of 35mph and is scheduled to travel 227.5 miles. How long will this take in hours and minutes?

$$Time = \frac{distance}{speed}$$

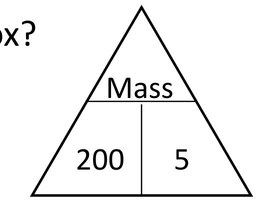
$$Time = \frac{227.5}{35} = 6.5 hours = 6\ hours\ 30\ minutes$$



A 5m^3 box has a density of 200g/m^3. What is the mass of the box?

$$Mass = Density\ \times Volume$$

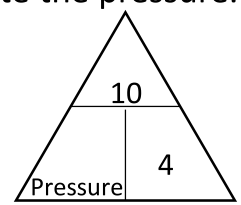
$$Mass = 200 \times 5 = 1000g$$



10N of force are applied to a block with area 4m^2. Calculate the pressure.

$$Pressure = \frac{force}{area}$$

$$Pressure = \frac{10}{4} = 2.5N/m^2$$



U151 U910 U527

Key Words

Speed
Distance
Time
Pressure
Force
Area
Density
Mass
Volume

1) A block exerts a force of 120 Newtons on the ground. The block has an area of 2 m^2.
Work out the pressure on the ground.

2) A piece of gold has a mass of 760 grams and a volume of 40 cm^3.
Work out the density of the piece of gold.

3) Dani leaves her house at 08 00. She drives 63 miles to work. She drives at an average speed of 27 miles per hour. At what time does Dani arrive at work?

ANSWERS 1) 60N/m^2 2) 19g/cm^3 3) 10:20am

ANGLE FACTS INCLUDING ON PARALLEL LINES

Key Concepts

Angles in a **triangle equal 180°**.

Angles in a **quadrilateral equal 360°**.

Vertically opposite angles are equal in size.

Angles on a **straight line equal 180°**.

Base angles in an isosceles triangle are equal.

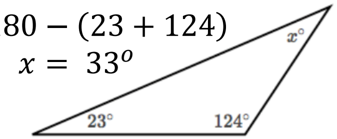
Alternate angles are equal in size.

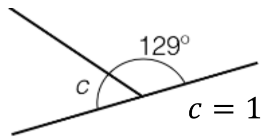
Corresponding angles are equal in size.

Allied/co-interior angles are equal 180°.

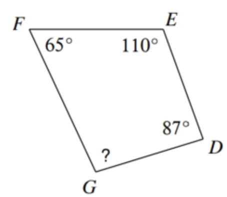
Examples

$x = 180 - (23 + 124)$
$x = 33^o$

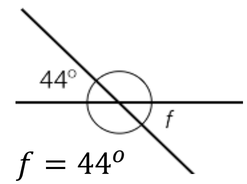




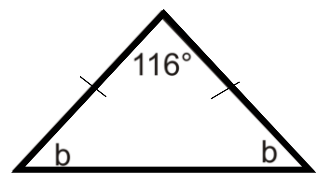
$c = 180 - 129$
$x = 51^o$



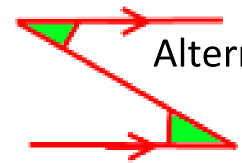
$? = 360 - (65 + 110 + 87)$
$? = 98^o$



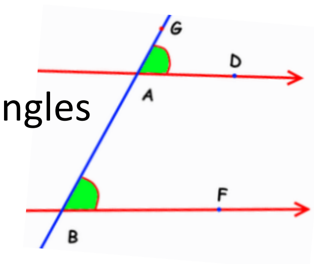
$f = 44^o$



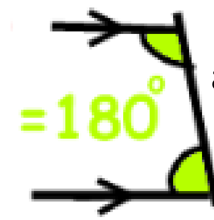
$b = (180 - 116) \div 2$
$b = 32^o$



Alternate angles are equal



Corresponding angles are equal



Allied/co-interior angles equal 180°

sparx

M331

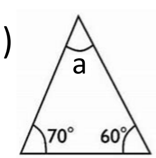
M606

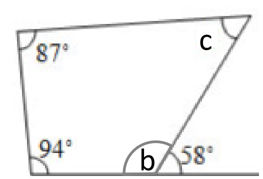
Key Words

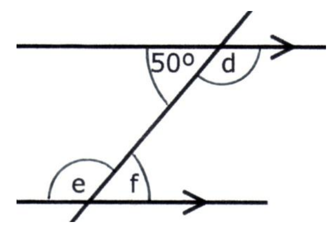
Angle
Vertically opposite
Straight line
Alternate
Corresponding
Allied
Co-interior

Questions

Calculate the missing angle:

a) 

b) 

c) 

ANSWERS: 1) a=50° 2) b=122° c=57° 3) d=130° e=130° f=50°

BEARINGS

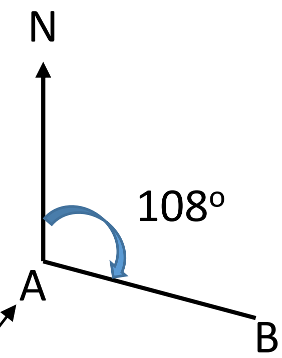
Key Concepts

Bearings are a type of angle that are used in real life directional instructions. They have **three rules** that they must conform to:

1) They must always be **measured from North**.
2) They must always be measured in a **clockwise direction**.
3) They must always have **3 figures** e.g. 72° is written as 072°

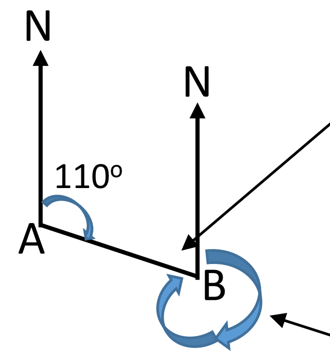
Examples

The bearing of B **from** A is 108°



Where we start measuring from using our **protractor**

We don't always need a protractor to find bearings, we can use our angle facts knowledge.



Because we know co-interior angles sum to 180°, this angle must be 70°.

The angle we are finding is the clockwise angle from B. We know angles around a point sum to 360°.

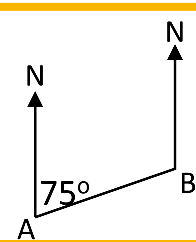
The bearing of A **from** B is 290°

sparx

M260, M416, U525, U107

Key Words

Bearing
Clockwise
North
Angle
Protractor

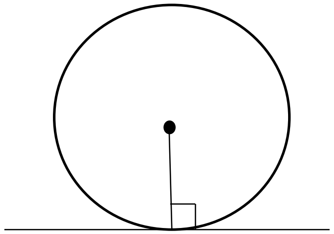


The bearing of B **from** A is 075°.
Calculate the bearing of A **from** B.

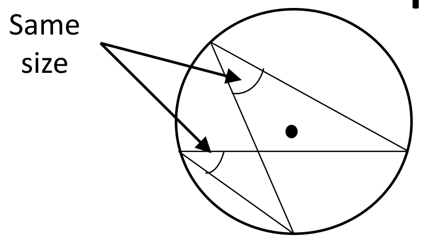
ANSWERS 255°

CIRCLE THEOREMS

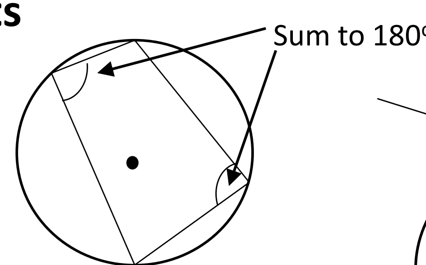
Key Concepts



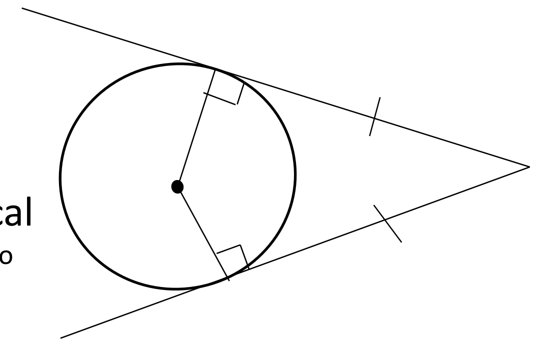
The angle between a radius and a tangent is 90°



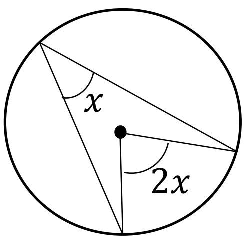
Angles at the circumference are equal



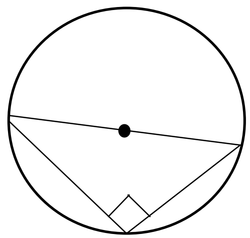
Opposite angles in a cyclical quadrilateral sum to 180°



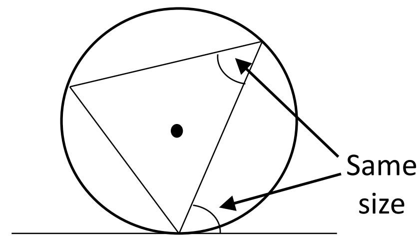
From any point you can only draw two tangents, and they are equal in length



The angle at the centre is twice that at the circumference



The angle in a semi circle is 90°



The alternate segment theorem

sparx

U251, U459, U130, U489, U808

Key Words

Radius
Centre
Tangent
Circumference
Right angle

Try look, cover, write, check to be able to identify and describe each of the 7 circle theorems.

1. Read through the theorems
2. Cover them over
3. Attempt to recreate them on another sheet of paper
4. Check how many you remembered perfectly. Try again until you have all 7.

ANSWER

PYTHAGORAS AND TRIGONOMETRY

Key Concepts

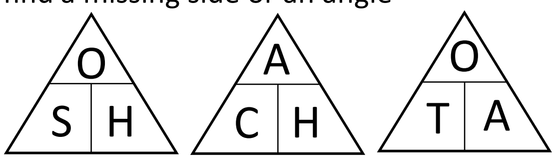
Pythagoras' theorem and basic trigonometry both work with **right angled triangles.**

Pythagoras' Theorem – used to find a missing length when two sides are known

$$a^2 + b^2 = c^2$$

c is always the hypotenuse (the longest side)

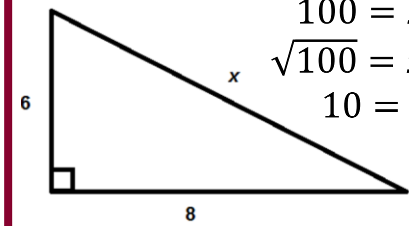
Basic trigonometry SOHCAHTOA – used to find a missing side or an angle



When finding the missing angle we must press **SHIFT** on our calculators first.

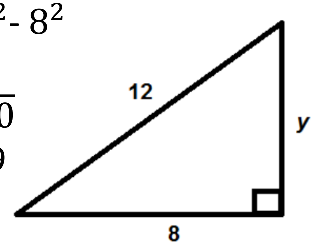
Examples

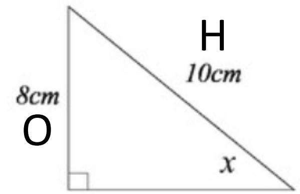
Pythagoras' Theorem

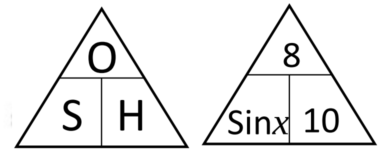


$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = x$$
$$10 = x$$

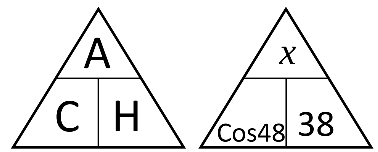
$$a^2 + b^2 = c^2$$
$$a^2 + 8^2 = 12^2$$
$$a^2 = 12^2 - 8^2$$
$$a^2 = 80$$
$$a = \sqrt{80}$$
$$a = 8.9$$



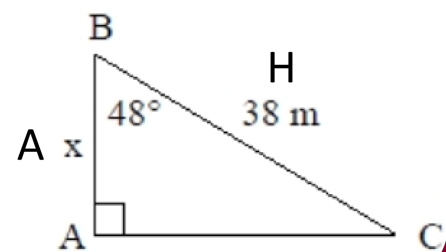




$$\sin x = \frac{8}{10}$$
$$x = \sin^{-1}\left(\frac{8}{10}\right)$$
$$x = 53.1^o$$



$$\cos 48 = \frac{x}{38}$$
$$38 \times \cos 48 = x$$
$$x = 25.4m$$



sparx

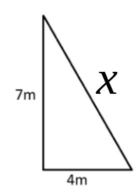
M677

Key Words

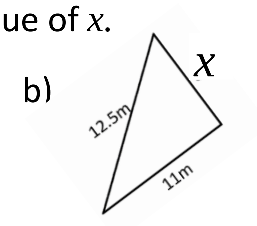
Right angled triangle
Hypotenuse
Opposite
Adjacent
Sine
Cosine
Tangent

Find the value of x.

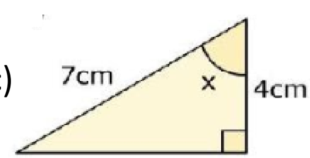
a)



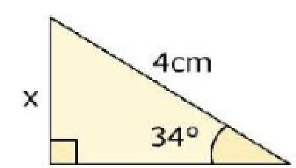
b)



c)



d)



ANSWERS: a) 8.06m b) 5.94m c) 55.15° d) 2.34cm

THE SINE AND COSINE RULE

Key Concepts

Sine rule

To calculate a missing side:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

To calculate a missing angle:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Cosine rule

To calculate a missing side:

$$a^2 = b^2 + c^2 - 2bccosA$$

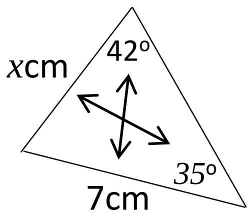
To calculate a missing angle:

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

Area of a triangle using sine

$$area = \frac{1}{2}absinC$$

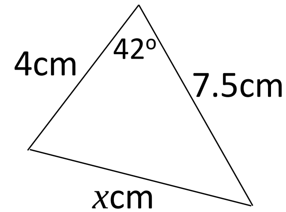
Examples



$$\frac{x}{sin35} = \frac{7}{sin42}$$

$$x = \frac{sin35 \times 7}{sin42}$$

$$x = 6.0cm$$

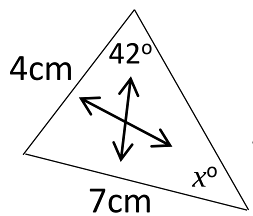


$$a^2 = b^2 + c^2 - 2bccosA$$

$$x^2 = 4^2 + 7.5^2 - 2 \times 4 \times 7.5 \times cos42$$

$$x^2 = 27.66$$

$$x = \sqrt{27.66} = 5.26cm$$

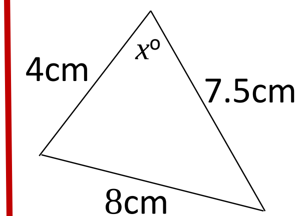


$$\frac{sinx}{4} = \frac{sin42}{7}$$

$$sinx = \frac{sin42 \times 4}{7}$$

$$x = sin^{-1}\left(\frac{sin42 \times 4}{7}\right)$$

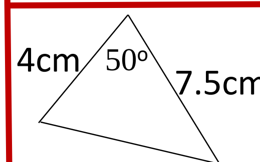
$$x = 22.5^o$$



$$cosA = \frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}$$

$$A = cos^{-1}\left(\frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}\right)$$

$$A = 82.1^o$$



$$area = \frac{1}{2} \times 4 \times 7.5 \times sin50$$

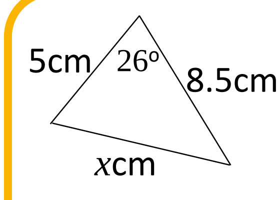
$$area = 11.49cm^2$$

sparx

U164

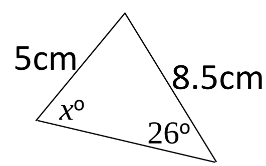
Key Words

Sine
Cosine
Side
Angle
Inverse
2D



1a) Calculate x

b) Calculate the area of the triangle



2a) Calculate x

b) Calculate the area of the triangle

ANSWERS 1a) 4.57cm b) 9.32cm² 2a) 48.18° b) 20.45cm²

SEQUENCES

Key Concepts

Arithmetic or linear sequences increase or decrease by a common amount each time.

Geometric series has a common multiple between each term.

Quadratic sequences include an n^2. It has a common second difference.

Fibonacci sequences are where you add the two previous terms to find the next term.

Linear/arithmetic sequence:

$$\overset{-3}{+1} \quad 4 \quad \overset{+3}{} \quad 7 \quad \overset{+3}{} \quad 10 \quad \overset{+3}{} \quad 13 \quad \overset{+3}{} \quad 16\ldots$$

a) State the nth term

$3n + 1$

Difference   The 0th term

b) What is the 100th term in the sequence?

$3n + 1$

$3 \times 100 + 1 = 301$

c) Is 100 in this sequence?

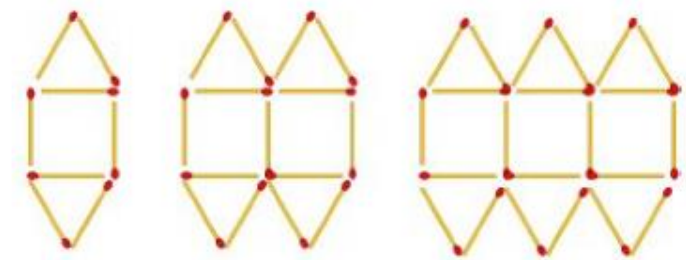
$3n + 1 = 100$

$3n = 99$

$n = 33$

Yes as 33 is an integer.

Pattern 1   Pattern 2   Pattern 3



Examples

Linear sequences with a picture:

State the nth term.

Hint: Firstly write down the number of matchsticks in each image:

$7n + 1$

Pattern 1	Pattern 2	Pattern 3
8	15	22

+1   -7   +7   +7

Geometric sequence e.g.

$4 \xrightarrow{\times 3} 12 \xrightarrow{\times 3} 36 \xrightarrow{\times 3} 108\ldots$

Quadratic sequence e.g. $n^2 + 4$ Find the first 3 numbers in the sequence

First term: $1^2 + 4 = 5$

Second term: $2^2 + 4 = 8$

Third term: $3^2 + 4 = 13$

sparx

M991, M418, M166, M981

Key Words

Linear
Arithmetic
Geometric
Sequence
Nth term

1) 1, 8, 15, 22, ….

a) Find the nth term   b) Calculate the 50th term   c) Is 120 in the sequence?

2) $n^2 - 5$ Find the first 4 terms in this sequence

ANSWERS: 1a) $7n - 6$ b) 344 c) 18 so yes as n is an integer 2) -4, -1, 4, 11

DIVIDING AN AMOUNT INTO RATIOS

Key Concepts

An amount can be divided into a given ratio.

Red : Green
1 : 3

For every 1 red there are 3 greens.

A ratio can be converted into fractions.

Red : Green
1 : 3

$\frac{1}{4}$ are red and $\frac{3}{4}$ are green.

Examples

A woman has £400. She is going to split her money between her two children in the ratio 2:3. How much does each child receive?

No. of boxes (2+3)

$400 \div 5$
$= 80$

2	:	3
$\boxed{80}$		$\boxed{80}$
$\boxed{80}$		$\boxed{80}$
£160		$\boxed{80}$
		£240

Child 1 receives £160 and Child 2 receives £240.

There are boys and girls at a party in the ratio 5:2.
There are 15 more boys than girls.
Calculate the number of people at the party.

No. of extra Boxes (5-2)

$15 \div 3$
$= 5$

5	:	2
$\boxed{5}$		$\boxed{5}$
$\boxed{5}$		$\boxed{5}$
$\boxed{5}$		
$\boxed{5}$		
$\boxed{5}$		

7×5
$= 35$ people

sparx

M885, M801, M267, M525,

Key Words

Ratio
Divide
Parts

1) Ann made some cakes. She made vanilla cakes and chocolate cakes in the ratio 2:9. What fraction of the cakes were chocolate?
2) Share £25 in the ratio 7:3
3) Katy and Becky share some money in the ratio 2:1. Katy receives £10 more than Becky. How much do they each receive?
4) Claire and John share some money in the ratio 3:2. Claire receives £18. How much does John receive?

ANSWERS 1) $\frac{9}{11}$ 2) £17.50, £7.50 3) £20, £10 4) £12