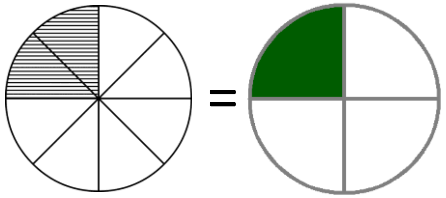
RATIO

Key Concept

2 parts → 2:6 ← 6 parts

=

1:3



$= \frac{1}{4}$

Key Words

Ratio: Relationship between two numbers.

Part: This is the numeric value '1' of, would be equivalent to.

Simplify: Divide both parts of a ratio by the same number.

Equivalent: Equal in value.

Convert: Change from one form to another.

Examples

Simplify 60 : 40 : 100

÷ 10

6 : 4 : 10

÷ 2

3 : 2 : 5

This could have been done in one step by dividing by 20.

Write 2: 5 in the form 1 : n

2 : 5

÷ 2 ÷ 2

1 : 2.5

Share £45 in the ratio 2 : 7

2 : 7

2	7
5	5
5	5
=10	5
	5
	5
	5
	5
	=35

45 ÷ 9 = 5

£10 : £35

Joy and Martin share money in the ratio 2 : 5. Martin gets £18 more than Joy. How much do they each get?

2 : 5

2	5
6	6
6	6
	6
	6
	6
=12	=30

18 ÷ 3 = 6

£12 : £30

sparx

M981,U171, U680

Tip

Its often useful to write the letters above the ratio. This helps you keep the order the correct way round.

Questions

1) Simplify a) 45 : 63 b) 66 : 44 c) 320 : 440
2) Write in the form 1 : n a) 5 : 10 b) 4: 6 c) $x : x^2 + x$
3) Share 64 in the ratio 3 : 5 4) Write the ratio 1 : 4 as a fraction.

ANSWERS: 1) a) 5 : 7 b) 3 : 2 c) 8 : 11 2) a) 1 : 2 b) 1 : 1.5 c) $1 : x + 1$ 3) 24 : 40 4) $^1/_5$

DIVIDING AN AMOUNT INTO RATIOS

Key Concepts

An amount can be divided into a given ratio.

Red : Green
1 : 3

For every 1 red there are 3 greens.

A ratio can be converted into fractions.

Red : Green
1 : 3

$\frac{1}{4}$ are red and $\frac{3}{4}$ are green.

Examples

A woman has £400. She is going to split her money between her two children in the ratio 2:3. How much does each child receive?

No. of boxes (2+3)

$400 \div 5$
$= 80$

2	:	3
$\boxed{80}$		$\boxed{80}$
$\boxed{80}$		$\boxed{80}$
£160		$\boxed{80}$
		£240

Child 1 receives £160 and Child 2 receives £240.

There are boys and girls at a party in the ratio 5:2.
There are 15 more boys than girls.
Calculate the number of people at the party.

No. of extra Boxes (5-2)

$15 \div 3$
$= 5$

5	:	2
$\boxed{5}$		$\boxed{5}$
$\boxed{5}$		$\boxed{5}$
$\boxed{5}$		
$\boxed{5}$		
$\boxed{5}$		

7×5
$= 35$ people

sparx

M885, M801, M267, M525,

Key Words

Ratio
Divide
Parts

1) Ann made some cakes. She made vanilla cakes and chocolate cakes in the ratio 2:9. What fraction of the cakes were chocolate?
2) Share £25 in the ratio 7:3
3) Katy and Becky share some money in the ratio 2:1. Katy receives £10 more than Becky. How much do they each receive?
4) Claire and John share some money in the ratio 3:2. Claire receives £18. How much does John receive?

ANSWERS 1) $\frac{9}{11}$ 2) £17.50, £7.50 3) £20, £10 4) £12

RATIO AND DIRECT PROPORTION

Key Concepts

To calculate the **value** for a single item we can use the **unitary method**.

When working with best value in monetary terms we use:

$$Price\ per\ unit = \frac{price}{quantity}$$

In recipe terms we use:

$$Weight\ per\ unit = \frac{weight}{quantity}$$

If 20 apples weigh 600g. How much would 28 apples weigh?

600 ÷ 20 = 30g weight of 1 apple
28 × 30 = **840g**

Box A has 8 fish fingers costing £1.40.
Box B has 20 fish fingers costing £ 3.40.
Which box is the better value?

$$A = \frac{£1.40}{8} = £0.175 \qquad B = \frac{£3.40}{20} = £0.17$$

Therefore Box B is better value as each fish finger costs less.

Examples

The recipe shows the ingredients needed to make 10 Flapjacks.
How much of each will be needed to make 25 flapjacks?

Ingredients for 10 Flapjacks

80 g rolled oats
60 g butter
30 m*l* golden syrup
36 g light brown sugar

Method 1: Unitary

80 ÷ 10 = 8
8 × 25 = **200g**

30 ÷ 10 = 3
3 × 25 = **75g**

60 ÷ 10 = 6
6 × 25 = **150g**

36 ÷ 10 = 3.6
3.6 × 25 = **90g**

Method 2: 5 flapjacks

80 ÷ 2 = 40
40 × 5 = **200g**

30 ÷ 2 = 15
15 × 5 = **75g**

60 ÷ 2 = 30
30 × 5 = **150g**

36 ÷ 2 = 18
18 × 5 = **90g**

sparx

U577
U753
U865

Key Words

Unitary
Best Value
Proportion
Quantity

Ingredients
to make 16 gingerbread men

180 g flour
40 g ginger
110 g butter
30 g sugar

1) How much will we need to make 24 gingerbread men?

2) Packet A has 10 toilet rolls costing £3.50.
Packet B has 12 toilet rolls costing £3.60.
Which is better value for money?

3) If 15 oranges weigh 300g. What will 25 oranges weigh?

ANSWERS 1) 270g flour, 60g ginger, 165g butter, 45g sugar 2) Packet B 30p per roll 3) 500g

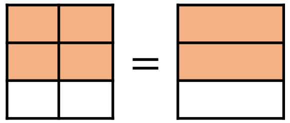
PROPORTION

Key Concept

Proportion states that two fractions or ratios are equivalent.

$$\frac{4}{6} = \frac{2}{3}$$

$$4:2 = 2:1$$



Key Words

Ratio: Relationship between two numbers.

Scale: The ratio of the length in a drawing to the length of the real thing.

Proportion: A name we give to a statement that two ratios are equal.

Exchange rate: The value of one currency for the purpose of conversion to another.

Examples

Write 2: 5 in the form 1 : n

$$2:5 \xrightarrow{\div 2 \ \div 2} 1:2.5$$

a:b = 4:5 and b:c = 6:7

Find a:b:c.

The LCM of 5 and 6 is 30

a	:	b	:	c
4	:	5		
		6	:	7
24	**:**	**30**	**:**	**35**

×6 ×5

Cake recipe for 6 people.

3 eggs

300g flour

150g sugar

What would you need for 8 people?

÷ 3 ×4

	6	**2**	**8**
eggs	3	1	4
flour	300g	100g	400g
sugar	150g	50g	200g

sparx

M543,U610,M229,M112,M260

Tip

Working with ratio or proportion requires multiplying or dividing the numbers. Do not add or subtract.

Questions

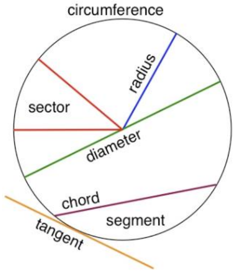
1) Write in the form 1 : n a) 4 : 8 b) 3 : 12 c) 4 : 6
2) a : b = 3 : 10 and b : c = 4 : 12. Find a:b:c.
3) Pancakes for 4 people need 2 eggs, 120g flour and 60ml milk. How much for 6 people?

ANSWERS: 1) a) 1:2 b) 1:4 c) 1:1.5 2) 12:40:120 3) 3 eggs, 180g flour, 90 ml milk.

PERIMETER AND CIRCUMFERENCE

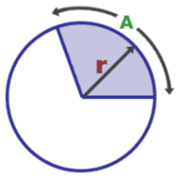
Key Concepts

Parts of a circle



Circumference of a circle is calculated by πd and is the distance around the circle.

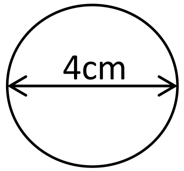
Arc length of a sector is calculated by $\frac{\theta}{360}\pi d.$



Examples

Calculate:

a) **Circumference**



C = $\pi \times 4$

= 4π

or = 12.57cm

b) **Diameter** when the circumference is 20cm

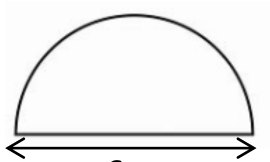
C = $\pi \times d$

20 = $\pi \times d$

$\frac{20}{\pi} = d$

Or $6.37cm$

c) **Perimeter**



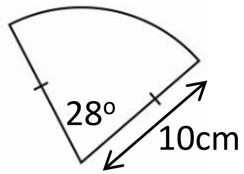
$P = \frac{\pi \times d}{2} + d$

$P = \frac{\pi \times 6}{2} + 6$

$P = 3\pi + 6$

Or = 15.42cm

d) **Arc length**



$\text{Arc} = \frac{\theta}{360} \times \pi \times d$

$\text{Arc} = \frac{28}{360} \times \pi \times 2 \times 10$

$\text{Arc} = \frac{28}{360} \times \pi \times 20$

$\text{Arc} = \frac{14}{9}\pi$

Or = 4.89cm

sparx

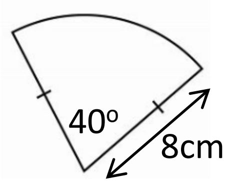
U604 U950 U221

Key Words

Circle
Perimeter
Circumference
Radius
Diameter
Pi
Arc

Calculate:

1) The circumference of a circle with a diameter of 12cm
2) The diameter of a circle with a circumference of 30cm
3) The perimeter of a semicircle with diameter 15cm
4) The arc length of the diagram



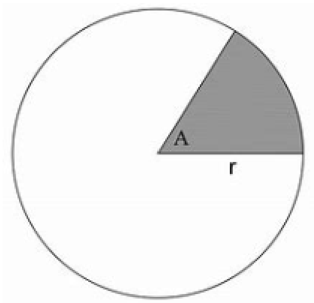
ANSWERS: 1) 12π or 37.7cm 2) $\frac{30}{\pi}$ or 9.54cm 3) 38.56cm 4) $\frac{16}{9}\pi$ or 5.59cm

AREA OF CIRCLES AND PART CIRCLES

Key Concepts

The **area** of a circle is calculated by πr^2

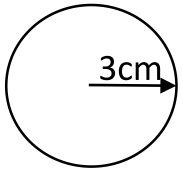
The **area of a sector** is calculated by $\frac{\theta}{360}\pi r^2$



Examples

Calculate:

a) **Area**



A = $\pi \times 3^2$
= 9π
or = 28.3cm^2

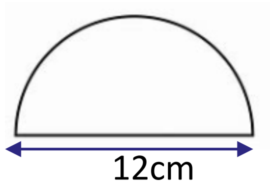
b) **Radius** when the area is 20cm^2

A = $\pi \times r^2$
20 = $\pi \times r^2$
$\frac{20}{\pi} = r^2$

$\sqrt{\frac{20}{\pi}} = r$

Or $2.52cm$

c) **Area**



$P = \frac{\pi \times r^2}{2}$

$P = \frac{\pi \times 6^2}{2}$

$P = 18\pi$

Or = 56.55cm^2

d) **Area of a sector**

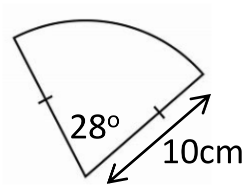
Arc $= \frac{\theta}{360} \times \pi \times r^2$

Arc $= \frac{28}{360} \times \pi \times 10^2$

Arc $= \frac{28}{360} \times \pi \times 100$

Arc $= \frac{70}{9}\pi$

Or = 24.43cm



sparx

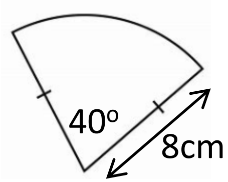
U604 U950
U221 U373

Key Words

Circle
Area
Radius
Diameter
Pi
Sector

Calculate:

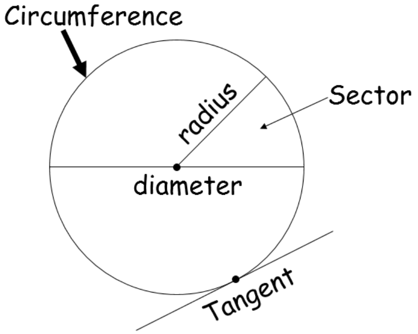
1) The area of a circle with a radius of 9cm
2) The radius of a circle with an area of 45cm^2
3) The area of a semicircle with diameter of 16cm
4) The area of the sector in the diagram

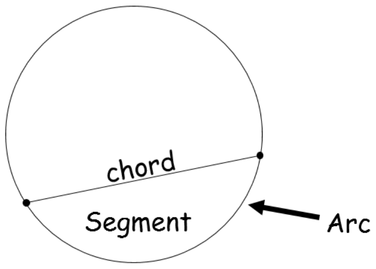


ANSWERS: 1) 81π or 254.47cm^2 2) $\sqrt{\frac{45}{\pi}}$ or 3.78cm^2 3) 32π or 100.53cm 4) $\frac{64}{9}\pi$ or 22.34cm

CIRCLES AND COMPOUND AREA

Key Concepts





Key Words

Diameter: Distance from one side of the circle to the other, going through the centre.

Radius: Distance from the centre of a circle to the circumference.

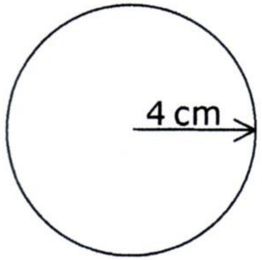
Chord: A line that intersects the circle at two points.

Tangent: A line that touches the circle at only one point.

Compound (shape): More than one shape joined to make a different shape.

Examples

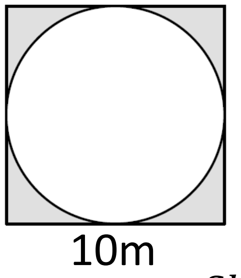
Find the area and circumference to 2dp.



$$Circumference = \pi \times d$$
$$= \pi \times 8 = 25.13cm$$

$$Area = \pi \times r^2$$
$$= \pi \times 4^2 = 50.27cm^2$$

Find shaded area to 2dp.



$$Square\ area = 10 \times 10$$
$$= 100m^2$$

$$Circle\ area = \pi \times r^2$$
$$= \pi \times 5^2$$
$$= 78.54\ m^2$$

$$Shaded\ area = 100 - 78.54 = 21.46m^2$$

sparx

M169,M231, M280,M291, M595

Tip

If you don't have a calculator you can leave your answer in terms of π.

Formula

$$Circle\ Area = \pi \times r^2$$
$$Circumference = \pi \times d$$

Questions

1) Find to 1dp the area and circumference of a circle with:

a) Radius = 5cm b) Diameter = 12mm c) Radius = 9m

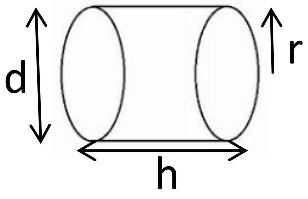
2) Find the area & perimeter of a semi-circle with diameter of 15cm.

ANSWERS: 1) a) A = 78.5cm^2, C = 31.4cm b) A = 113.1mm^2, C = 37.7mm c) A = 254.5m^2, C = 56.5m 2) A = 88.4cm^2, P = 38.6cm

VOLUME AND SURFACE AREAS OF CYLINDERS

Key Concepts

A **cylinder** is a **prism** with the cross section of a circle.

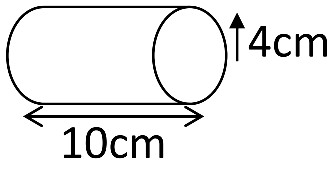


The **volume** of a cylinder is calculated by $\pi r^2 h$ and is the space inside the 3D shape

The **surface area** of a cylinder is calculated by $2\pi r^2 + \pi dh$ and is the total of the areas of all the faces on the shape.

Examples

From the diagram calculate:



a) **Volume**

$V = \pi \times r^2 \times h$

$V = \pi \times 4^2 \times 10$

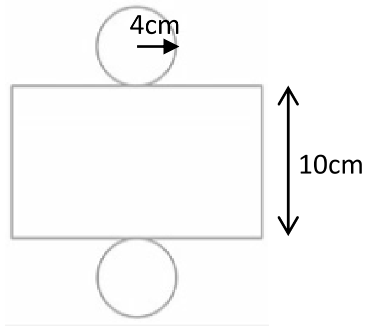
$V = 160\pi$

Or $= 502.65cm^3$

b) **Surface Area –** You can use the net of the shape to help you

$Area\ of\ two\ circles$
$= 2 \times \pi \times r^2$
$= 2 \times \pi \times 4^2$
$= 32\pi$

$Area\ of\ rectangle$
$= \pi \times d \times h$
$= \pi \times 8 \times 10$
$= 80\pi$



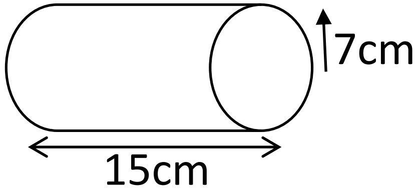
$Surface\ Area = 32\pi + 80\pi$
$= 112\pi$
or $= 351.86cm^3$

sparx

M915
U163

Key Words
Cylinder
Surface Area
Radius
Diameter
Pi
Volume
Prism

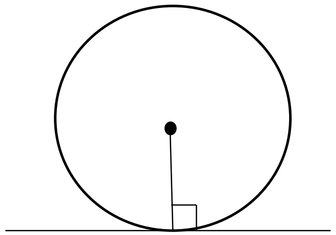
Calculate the volume and surface area of this cylinder



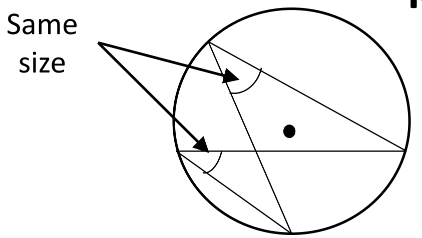
ANSWERS: Volume = 735π or $2309.07cm^3$ Surface area = 308π or $967.61cm^3$

CIRCLE THEOREMS

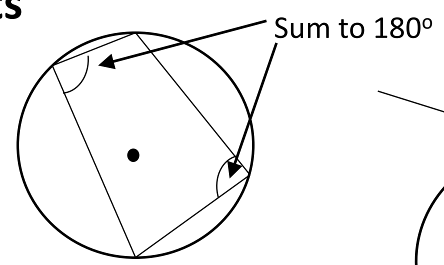
Key Concepts



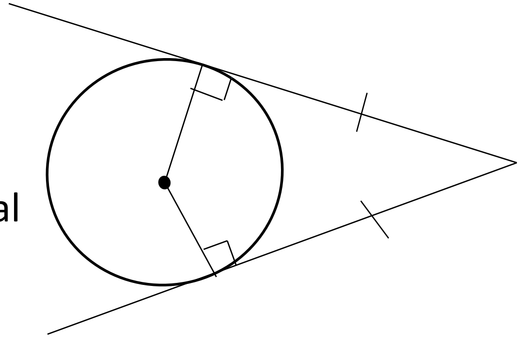
The angle between a radius and a tangent is 90°



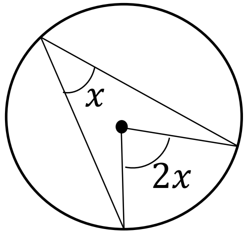
Angles at the circumference are equal



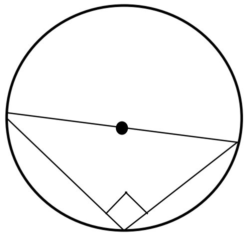
Opposite angles in a cyclical quadrilateral sum to 180°



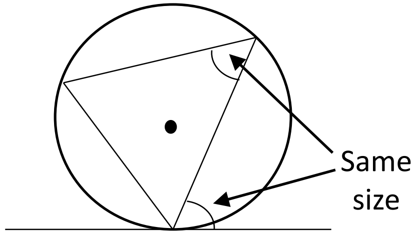
From any point you can only draw two tangents, and they are equal in length



The angle at the centre is twice that at the circumference



The angle in a semi circle is 90°



The alternate segment theorem

sparx

U251, U459, U130, U489, U808

Key Words
Radius
Centre
Tangent
Circumference
Right angle

Try look, cover, write, check to be able to identify and describe each of the 7 circle theorems.

1. Read through the theorems
2. Cover them over
3. Attempt to recreate them on another sheet of paper
4. Check how many you remembered perfectly. Try again until you have all 7.

ANSWER

TANGENT TO A CIRCLE

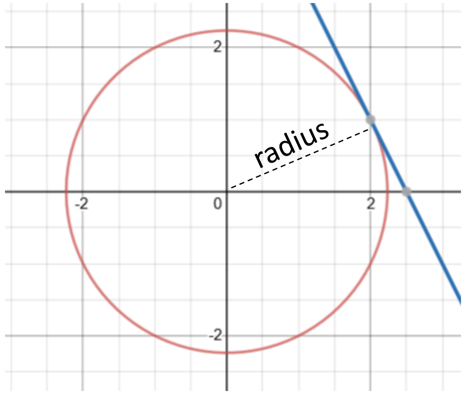
Key Concepts

A **tangent** touches a circle at **one point**.

A **tangent** line is **perpendicular** to the **radius** of the circle.

The gradient of the tangent is the **negative reciprocal** of the gradient of the equation of the line of the radius.

Examples



Find the equation of the tangent to the circle with equation:

$$x^2 + y^2 = 5$$

which passes through the point (2,1).

1) Find the equation of the line which is the radius of the circle.

$$gradient = \frac{1}{2} \quad therefore\ y = \frac{1}{2}x$$

2) The tangent is perpendicular to the radius.

$$gradient\ of\ tangent = negative\ reciprocal\ of\ \frac{1}{2}$$
$$= -2$$

3) Substitute in the given coordinate (2,1) to $y = -2x + c$

$$y = -2x + c$$
$$1 = (-2 \times 2) + c$$
$$1 + 4 = c$$
$$5 = c$$
$$y = -2x + 5$$

sparx

U567

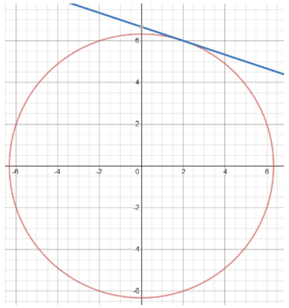
Key Words

Radius
Tangent
Negative reciprocal
Perpendicular
Gradient

Find the equation of the tangent to the circle with equation:

$$x^2 + y^2 = 40$$

which passes through the point (2,6).



ANSWER $y = -\frac{1}{3}x + \frac{20}{3}$

EQUATION OF A CIRCLE

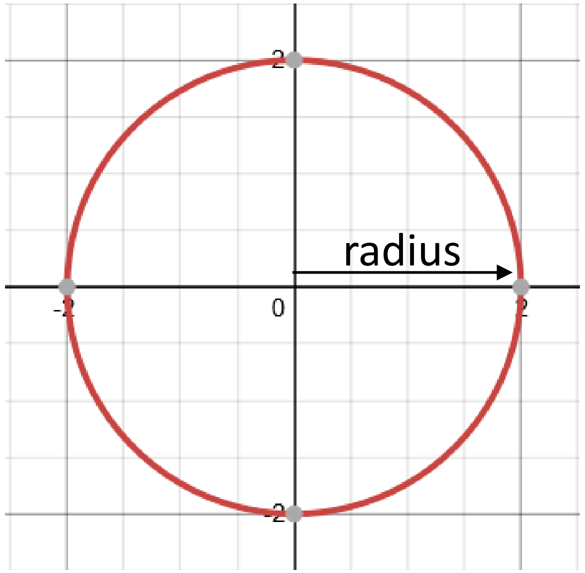
Key Concepts

The **equation of a circle** will be in the format:

$$x^2 + y^2 = radius^2$$

The **centre** of each circle will be at the coordinate **(0,0).**

Examples



$$x^2 + y^2 = 4$$

$$\begin{aligned} Radius &= \sqrt{4} \\ &= \pm 2 \end{aligned}$$

Therefore we can plot the following coordinates to support us sketching our graph: (0,2), (0,-2), (2,0), (-2,0)

sparx

U567

Key Words

Radius
Centre
Sketch
Square root

Calculate the length of the radius for each of the following equations of circles:

1) $x^2 + y^2 = 25$

2) $x^2 + y^2 = 49$

3) $x^2 + y^2 = 256$

4) $x^2 + y^2 = 22$

ANSWER 1) 5 2) 7 3) 16 4) $\sqrt{22}$

THEORETICAL PROBABILITY

Key Concepts

Probabilities can be described using **words** and **numerically**.

We can use **fractions, decimals or percentages** to represent a probability.

Theoretical probability is what should happen if all variables were fair.

All probabilities must **add to 1**.

The probability of something **NOT** happening equals:

$1 - (probability\ of\ it\ happening)$

Examples

Probability scale:

Impossible		Even chance		Certain
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3	5	2

1) What is the probability that a blue counter is chosen? $\frac{3}{19} = \frac{number\ of\ blue}{total\ number\ of\ counters}$
2) What is the probability that red is **not** chosen?
$\frac{10}{19} = \frac{number\ of\ all\ other\ colours}{total\ number\ of\ counters}$

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5
= 11

Probability of choosing black = $\frac{11}{100}$

sparx

U803 U408 U510

Key Words
Theoretical
Probability
Fraction
Decimal
Percentage
Certain
Impossible
Even chance

	1	**2**	**3**
Prob	5	4	9

1a) Calculate the probability of choosing a 2.
b) Calculate the probability of not choosing a 3.

	1	**2**	**3**
Prob	0.37	$2x$	x

2) Calculate the probability of choosing a 2 or a 3.

ANSWERS: 1a) $\frac{4}{18}$ b) $\frac{9}{18}$ 2) P(2) = 0.42 P(3) = 0.21

TWO WAY TABLES AND PROBABILITY TABLES

Key Concepts

Two way tables are used to tabulate a number of pieces of information.

Probabilities can be formulated easily from two way tables.

Probabilities can be written as a **fraction, decimal or a percentage** however we often work with fractions. You do not need to simplify your fractions in probabilities.

Estimating the number of times an event will occur

Probability × no. of trials

Examples

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5

= 11

Probability of choosing black = $\frac{11}{100}$

80 children went on a school trip. They went to London or to York.
23 boys and 19 girls went to London. 14 boys went to York.

	London	York	Total
Girls	19	**24**	**43**
Boys	23	14	**37**
Total	**42**	**38**	80

What is the probability that a person is chosen that went to London? $\frac{42}{80}$

If a girl is chosen, what is the probability that she went to York? $\frac{24}{38}$

sparx

U683

U981

Key Words

Two way table

Probability

Fraction

Outcomes

Frequency

	1	2	3
Prob	0.37	$2x$	x

1a) Calculate the probability of choosing a 2 or a 3.

b) Estimate the number of times a 2 will be chosen if the experiment is repeated 300 times.

2a) Complete the two way table:

	Year Group			Total
	9	10	11	
Boys			125	407
Girls		123		
Total	303	256		831

b) What is the probability that a Y10 is chosen, given that they are a girl .

ANSWERS: 1) a) P(2) = 0.42 P(3) = 0.21 b) 0.42×300=126 2b) 123/424

PROBABILITY TREE DIAGRAMS

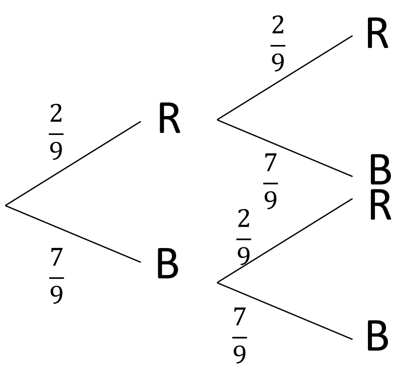
Key Concepts

Independent events are events which do not affect one another.

Dependent events affect one another's probabilities. This is also known as **conditional probability**.

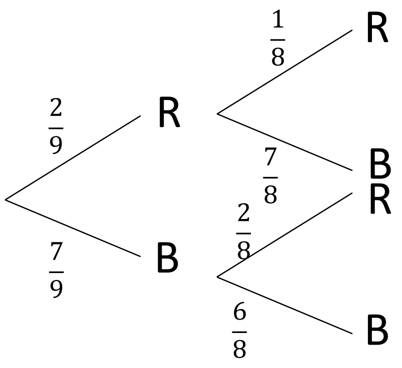
Examples

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:
$$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$
Prob of two blues :
$$\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$$
Prob of same colours:
$$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$$

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **not replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:
$$\frac{2}{9} \times \frac{1}{8} = \frac{2}{72}$$
Prob of two blues :
$$\frac{7}{9} \times \frac{6}{8} = \frac{42}{72}$$
Prob of same colours:
$$\frac{2}{72} + \frac{42}{72} = \frac{44}{72}$$

sparx

U558, U729, U821, U806

Key Words
Independent
Dependant
Conditional Probability
Fraction

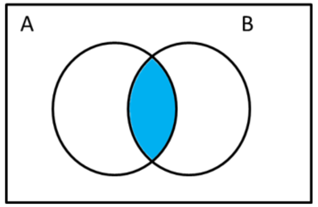
1) There are blue and green pens in a drawer. There are 4 blues and 7 greens.
A pen is chosen and then **replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

2) There are blue and green pens in a drawer. There are 4 blues and 7 greens.
A pen is chosen and **not replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

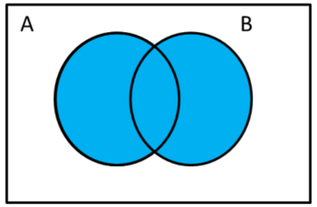
ANSWERS: 1) 56/121 2) 56/110

FURTHER PROBABILITY

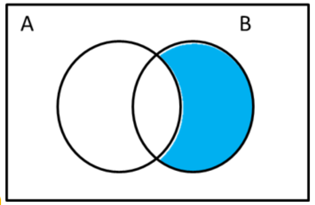
Key Concept



P(A ∩ B)



P(A ∪ B)



P(A′ ∩ B)

sparx

M718, M419
M829, M460

Key Words

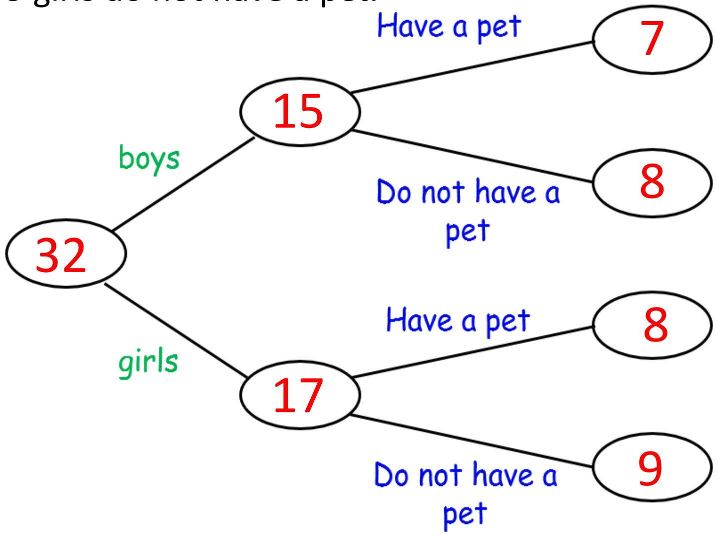
Probability: The chance of something happening as a numerical value.
Impossible: The outcome cannot happen.
Certain: The outcome will definitely happen.
Even chance: The are two different outcomes each with the same chance of happening.
Mutually Exclusive: Two events that cannot both occur at the same time.

Formula

$P(A \cap B) = P(A) \times P(B)$
$P(A \cup B) = P(A) + P(B)$
$or \quad (non\ ME) \quad P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$

Examples

In Hannah's class there are 32 students.
15 of these students are boys.
7 of the boys have a pet.
9 girls do not have a pet.



$$P(boy) = \frac{15}{32}$$

$$P(Girl\ with\ pet) = \frac{8}{32}$$

Questions

1) Draw a two-way table for the question above.
2) Find the probability that a pupil chosen is a boy with no pets.
3) A girl is chosen, what is the probability she has a pet?

ANSWERS: 2) $\frac{8}{32}$ 3) $\frac{8}{17}$