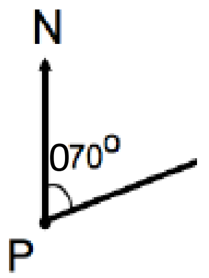
SCALES AND BEARINGS

Key Concepts

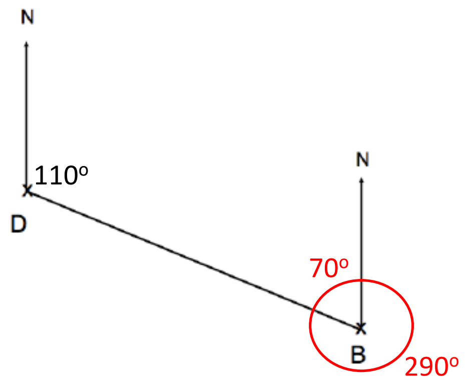
Scales are used to reduce real world dimensions to a useable size.

A **bearing** is an angle, measured **clockwise** from the **north** direction. It is given as a **3 digit** number.



Examples

The diagram shows the position of a boat B and dock D.



The scale of the diagram is 1cm to 5km.

a) Calculate the real distance between the boat and the dock.

$6cm = 6 \times 5$

$= 30km$

b) State the bearing of the boat from the dock.

110^o

c) Calculate the bearing of the dock from the dock.

$180^o - 110^o = 70^o$ because the angles are cointerior

$360^o - 70^o = 290^o$ because angles around a point equal 360°

sparx

U820, U525, U107

Key Words

Scale

Bearing

Clockwise

North

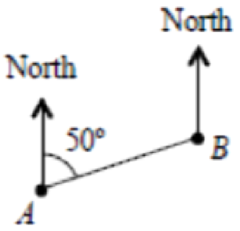
Links

Geography

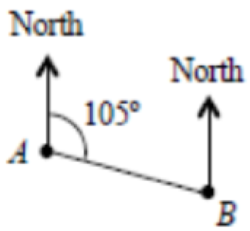
Find the bearing of A from B

(Diagrams not drawn to scale):

a)



b)



ANSWERS: (a) 230° (b) 285°

UNDERSTANDING PERCENTAGES and FRACTIONS

Key Concept

FDP equivalence

F	D	P
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Integer: Whole number.

Ascending Order: Place in order, smallest to largest.

Descending Order: Place in order, largest to smallest.

Examples

Make the denominators the same.

$\frac{3}{4} \downarrow \frac{6}{8} \downarrow \frac{1}{4}$ | 4 | 2 | $\frac{3}{8} \downarrow \frac{3}{8} \downarrow \frac{3}{8}$ | 3 | 5 | $\frac{1}{2} \downarrow \frac{4}{8} \downarrow \frac{1}{2}$ | 1

Convert them all to decimals.

56%	$\frac{3}{4}$	0.871	23%	$\frac{6}{7}$
0.56	0.75	0.871	0.23	0.857...
2	3	5	1	4
23%	56%	$\frac{3}{4}$	$\frac{6}{7}$	0.871

sparx

M429, M152, M803, M001, M835, M937, M437

Tip

- A larger denominator does not mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

Questions

1) Place these lists in ascending order.

a) $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{12}$ b) $\frac{3}{7}, \frac{1}{2}, 0.49, 0.2$ c) $\frac{7}{32}, 25\%, 0.05, \frac{29}{100}$

ANSWERS: 1) $\frac{7}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$ 2) $0.2, \frac{3}{7}, 0.49, \frac{1}{2}$ 3) $0.05, \frac{7}{32}, 25\%, \frac{29}{100}$

FRACTIONS, DECIMALS AND PERCENTAGES

Key Concepts

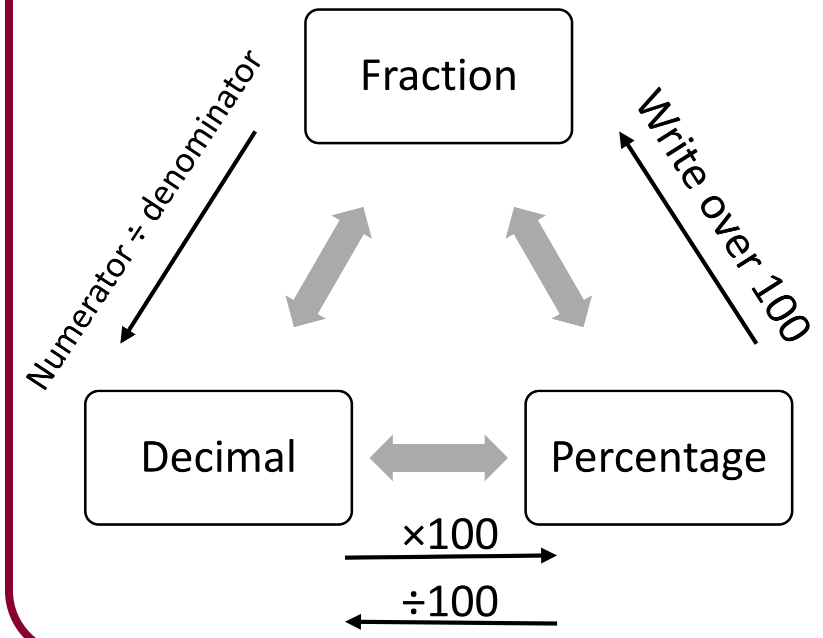
A **fraction** is a numerical quantity that is not a whole number.

A **decimal** is a number written using a system of counting based on the number 10.

Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
8	7	6	5	.	4	3	2

A **percentage** is an amount out of 100.

Examples



Order the following in ascending order:

$\frac{3}{5}$ 62% 0.67 $\frac{7}{10}$ 0.665

$\frac{3}{5} \xrightarrow{\times 20} \frac{60}{100} \rightarrow 60\%$

$62\% \rightarrow 62\%$

$0.67 \xrightarrow{\times 100} 67\%$

$\frac{7}{10} \xrightarrow{\times 10} \frac{70}{100} \rightarrow 70\%$

$0.665 \xrightarrow{\times 100} 66.5\%$

60% 62% 67% 70% 66.5%

$\frac{3}{5}$ 62% 0.665 0.67 $\frac{7}{10}$

sparx

M958

M264

M922

Key Words

Fraction

Decimal

Percentage

Division

Multiply

1) Convert the following into percentages:

a) 0.4 b) 0.08 c) $\frac{6}{20}$ d) $\frac{3}{25}$

2) Compare and order the following in ascending order:

$\frac{3}{4}$ 76% 0.72 $\frac{4}{5}$ 0.706

ANSWERS 1a) 40% b) 8% c) 30% d) 12% 2) 0.706 0.72 $\frac{3}{4}$ 76% $\frac{4}{5}$

PERCENTAGES

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Calculating percentages of an amount with a calculator:

Amount × percentage as a decimal

Calculating percentage increase/decrease:

Amount × (1 ± percentage as a decimal)

Examples

Calculating a percentage – non calculator:

Calculate 32% of 500g:

10% → 500 ÷ 10 = 50
30% → 50 × 3 = 150
1% → 500 ÷ 100 = 5
2% → 5 × 2 = 10

32% = 150 + 10 = 160g

Calculating a percentage – calculator:

Calculate 32% of 500g:

$Value \times (percentage \div 100)$
= 500×0.32
= 160g

Percentage change:

A dress is reduced in price by 35% from £80. What is it's **new price**?

$Value \times (1 - percentage\ as\ a\ decimal)$
= $80 \times (1 - 0.35)$
= £52

A house price appreciates by 8% in a year. It originally costs £120,000, what is the **new value** of the house?

$Value \times (1 + percentage\ as\ a\ decimal)$
= $120{,}000 \times (1 + 0.08)$
= £129,600

sparx

M433, M905, M476, M533

Key Words
Percent
Increase/decrease
Appreciate
Depreciate
Multiplier
Divide

1) Write the following as a decimal multiplier: a) 45% b) 3% c) 2.7%
2) Calculate 43% of 600 without using a calculator
3) Calculate 72% of 450 using a calculator
4a) Decrease £500 by 6%
b) Increase 65g by 24%
c) Increase 70m by 8.5%

ANSWERS 1a) 0.45 b) 0.03 c) 0.027 2) 258 3) 324 4a) £470 b) 80.6g c) 75.95m

PERCENTAGES AND INTEREST

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Per annum is often used in monetary questions meaning **per year.**

Depreciation means that the value of something is going down or reducing.

Examples

Simple interest:

Joe invest £400 into a bank account that pays 3% **simple interest** per annum.
Calculate how much money will be in the bank account after 4 years.

3% = £4 × 3
= £12
4 years = £12 × 4
Interest = £48
Total in bank account = £400 + £48
= £448

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum.
Calculate how much money will be in the bank account after 4 years.

$Value \times (1 \pm percentage\ as\ a\ decimal)^{years}$
$= 400 \times (1 + 0.03)^4$
$= 400 \times (1.03)^4$
= £450.20

sparx
M901

Key Words
Percent
Depreciate
Interest
Annum
Simple
Compound
Multiplier

1) Calculate a) 32% of 48 b) 18% of 26
2) Kane invests £350 into a bank account that pays out simple interest of 6%. How much will be in the bank account after 3 years?
3) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?

ANSWERS A 1a) 15.36 b) 4.68 2) £413 3) £724.67

PERCENTAGE CHANGE AND REVERSE PERCENTAGES

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Calculating percentages of an amount with a calculator:

Amount × percentage as a decimal

Calculating percentage increase/decrease:

Amount × (1 ± percentage as a decimal)

Examples

Percentage change:

A dress is reduced in price by 35% from £80. What is it's **new price**?

$Value \times (1 - percentage\ as\ a\ decimal)$
= $80 \times (1 - 0.35)$
= £52

A house price appreciates by 8% in a year. It originally costs £120,000, what is the **new value** of the house?

$Value \times (1 + percentage\ as\ a\ decimal)$
= $120{,}000 \times (1 + 0.08)$
= £129,600

Reverse percentages: This is when we are trying to find out the original amount.

A pair of trainers cost £35 in a sale. If there was 20% off, what was the **original price** of the trainers?

$Value \div (1 - 0.20)$
= $35 \div 0.8$
= £43.75

A vintage car has increased in value by 5%, it is now worth £55,000. What was it worth **originally**?

$Value \div (1 + 0.05)$
= $55{,}000 \div 1.05$
= £52,380.95

sparx

U349 U773
U671 U286

Key Words
Percent
Increase/decrease
Reverse
Multiplier
Inverse

1a) Decrease £500 by 6%
b) Increase 70 by 8.5%
2) A camera costs £180 in a 10% **sale**. What was the **pre-sale** price
3) The cost of a holiday, including **VAT** at 20% is £540. What is the **pre-VAT** price?

ANSWERS A 1a) £470 b) £75.95 2) £200 3) £450

COMPOUND INTEREST AND DEPRECIATION

Key Concepts

We use **multipliers** to increase and decrease an amount by a particular percentage.

Percentage increase:
$Value \times (1 + percentage\ as\ a\ decimal)$

Percentage decrease:
$Value \times (1 - percentage\ as\ a\ decimal)$

Appreciation means that the value of something is going up or increasing.

Depreciation means that the value of something is going down or reducing.

Per annum is often used in monetary questions meaning **per year.**

Examples

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum. Calculate how much money will be in the bank account after 4 years.

$Value \times (1 + percentage\ as\ a\ decimal)^{years}$
$= 400 \times (1 + 0.03)^4$
$= 400 \times (1.03)^4$
= £450.20

Compound depreciation:

The original value of a car is £5000. The value of the car **depreciates** at a rate of 7.5% per annum. Calculate the value of the car after 3 years.

$Value \times (1 - percentage\ as\ a\ decimal)^{years}$
$= 5000 \times (1 - 0.075)^3$
$= 5000 \times (0.925)^3$
= £3957.27

sparx

U773, U533, U332, U988

Key Words

Percent
Appreciate
Depreciate
Interest
Annum
Compound
Multiplier

1) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?
2) A house has decreased in value by 3% for the past 4 years. If originally it was worth £180,000, how much is it worth now?

ANSWERS A 1) £724.67 2) £159352.71

COLUMN VECTORS

Key Concepts

Vectors describe translations.

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

x: + move right, - move left

y: + move up, - move down

Examples

Adding vectors:

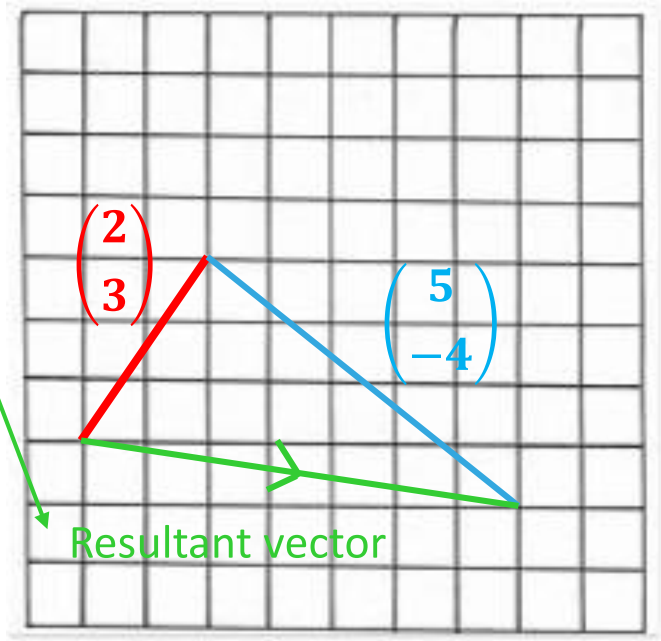
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} 2+5 \\ 3+-4 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

Subtracting vectors:

$$\begin{pmatrix} 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 3-2 \\ 9--3 \end{pmatrix} = \begin{pmatrix} 1 \\ 12 \end{pmatrix}$$

Vectors and scalar multipliers:

$$2\begin{pmatrix} 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \times 8 \\ 2 \times -3 \end{pmatrix} = \begin{pmatrix} 16 \\ -6 \end{pmatrix}$$



sparx

U632, U903, U564

Key Words

Column

Vector

Translation

Resultant

Calculate the resultant vector:

$a)\ \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -7 \end{pmatrix}$ $\quad b)\ \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\quad c)\ 3\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

ANSWER $a)\ \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ $\ b)\ \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\ c)\ \begin{pmatrix} 9 \\ -6 \end{pmatrix}$

TRANSLATION AND ENLARGEMENT

Key Concepts

A **translation** moves a shape on a coordinate grid. Vectors are used to instruct the movement:

$\begin{pmatrix} x \\ y \end{pmatrix}$

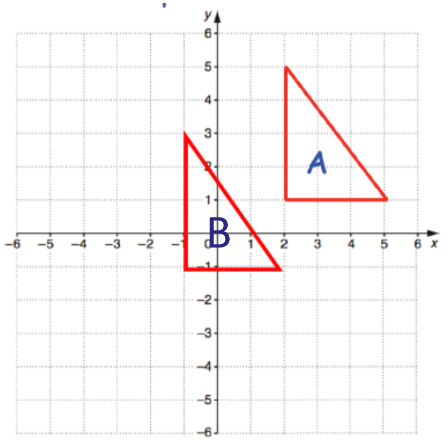
x: Positive-Right, Negative - Left

y: Positive-Up, Negative - Down

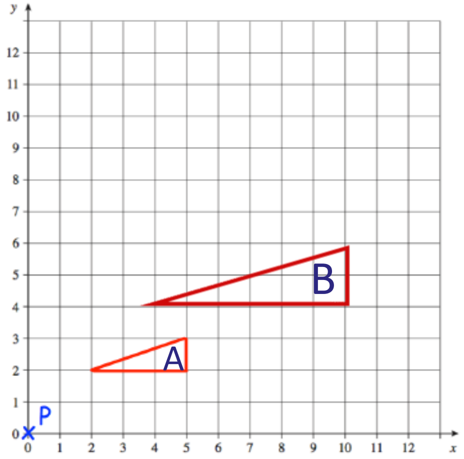
An **enlargement** changes the size of an image using a scale factor from a given point.

Examples

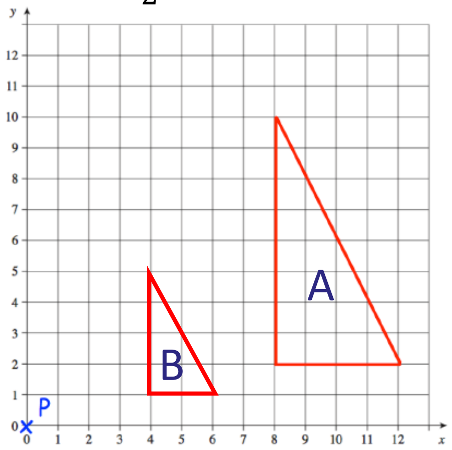
Translate shape A by $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$. Label it B



Enlarge shape A by scale factor 2 from point P.



Enlarge shape A by scale factor $\frac{1}{2}$ from point P.



sparx

U196

U519

U134

Key Words

Translation

Enlargement

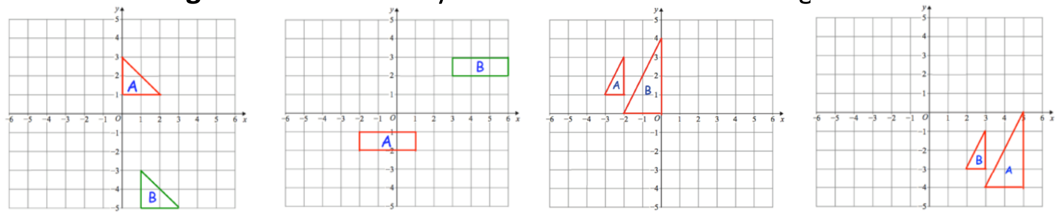
Scale factor

Centre

Positive

Negative

Describe the **single** transformation you see on each coordinate grid from A to B:



ANSWERS: a) translation $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ b) translation $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ c) enlarge, centre (-4,2) scale factor 2

d) enlarge, centre (1,-2) scale factor $\frac{1}{2}$

VECTORS IN DIAGRAMS

Key Concepts

Vectors notation:

$\boldsymbol{a}$ $\overrightarrow{AB}$ $\underline{\mathbf{a}}$

Magnitude: Length of the arrow

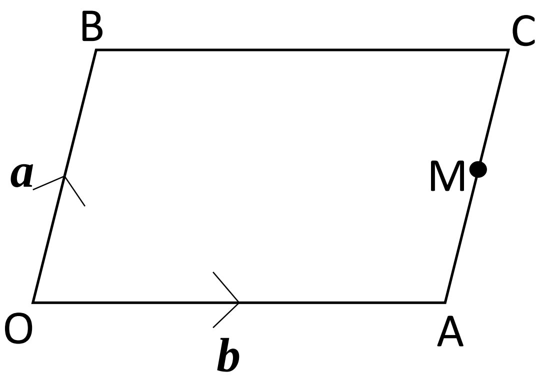
Direction: Where the arrow is pointing

Parallel lines of **equal length** have the **same** vector.

Parallel lines of **different lengths** have a **multiple** of the vector.

Travelling against an arrow **changes the sign** of the vector.

Examples



$\overrightarrow{OA} = \boldsymbol{b}$ $\overrightarrow{OB} = \boldsymbol{a}$

OABC is a parallelogram. M is the midpoint of AC.

a) State the vector of $\overrightarrow{OC}$.

As BC is parallel and equal in length to OA, it has the vector value of $\boldsymbol{b}$.

Therefore $\overrightarrow{OC} = \boldsymbol{a} + \boldsymbol{b}$

b) State the vector of $\overrightarrow{AO}$.

As we are travelling against the arrow, the vector changes sign.

Therefore $\overrightarrow{AO} = -\boldsymbol{b}$

c) State the vector of $\overrightarrow{OM}$.

As $\overrightarrow{AC}$ is parallel and equal in length to OB, is has the vector value of $\boldsymbol{a}$. M is the midpoint of $\overrightarrow{AC}$.

Therefore $\overrightarrow{OM} = \boldsymbol{b} + \frac{1}{2}\boldsymbol{a}$

sparx

U903, U564

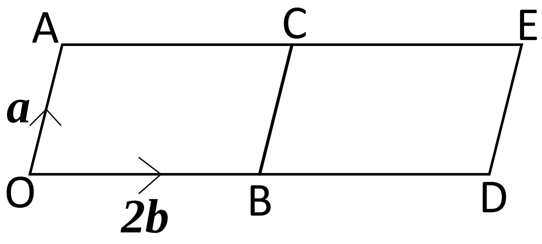
Key Words

Vector

Magnitude

Direction

Parallel



OABC and BCDE are two identical parallelograms.

a) State the vector of $\overrightarrow{OD}$

b) State the vector of $\overrightarrow{OC}$

c) State the vector of $\overrightarrow{AB}$

d) State the vector of $\overrightarrow{OE}$

ANSWER a) $4b$ b) $a + 2b$ c) $-a + 2b$ d) $4b + a$

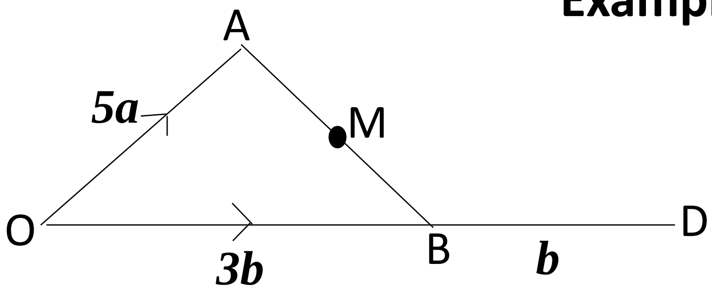
RATIO AND COLLINEAR PROOFS IN VECTORS

Key Concepts

Parallel lines of **different lengths** have a **multiple** of the vector.

For two vectors to form a **straight line** they must have vector values which are **multiples of one another** and must have a **common point**.

Examples



C is the point such that OC:CA = 4:1
M is the midpoint of AB.
D is the point such that OB:OD = 3:4
Show that C, M and D are on the same straight line.

$$\overrightarrow{CA} = \frac{1}{5}\overrightarrow{OA} = \frac{1}{5}(5a) = a$$

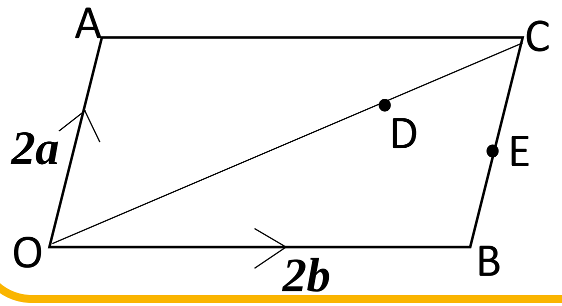
$$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = a + \frac{1}{2}(-5a + 3b) = a - 2.5a + 1.5b = -1.5a + 1.5b$$

$$\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BD} = \frac{1}{2}(-5a + 3b) + 4b = -2.5a + 1.5b + b = -2.5a + 2.5b$$

C, M and D are on a **straight line** as CM and MD are *multiples* of one another and have the **common point** of M.

U781, U660, U560, U781

Key Words
Vector
Ratio
Midpoint
Multiples



D is the point on OC such that OD:DC = 2:1.
E is the midpoint of BC.
Show that A, D and E are on the same straight line.

ANSWER: AD = $\frac{-2}{3}a + \frac{4}{3}b$ DE = $\frac{2}{3}b - \frac{1}{3}a$ *therefore A,D, and E are in a straight line as they are multiples of one another and have the common point of D.*