## PRESENTING AND INTERPRETTING DATA

## Key Concept

 Pie ChartsThere are 360 degrees in a pie chart. So you need angles that add to $360^{\circ}$.

| Eye colour | F |  |
| :---: | :---: | :---: |
| Blue | 15 | $\times 4=60$ |
| Brown | 43 | $\times 4=172$ |
| Other | 32 | $\times 4=128$ |
| $\frac{360}{90}=4$ | $=90$ | $=360$ |

Key Words
Frequency: Total.
Mean: Total of data divided by the number of pieces of data.
Mode: The value that occurs most frequently. Median: Middle number when they are in order.
Range: Difference between the largest and smallest values.

## Tips

- There can be more than one mode.
- Range is a measure of spread, not an average.
- Bar charts have gaps between the bars.


## Examples

$$
5,9,9,9,1112,13,15,16
$$

## Averages

$$
\text { Mean }=\frac{5+9+9+9+11+12+13+15+16}{9}=\frac{99}{9}=11
$$

Median = 11 (The middle number shown above)
Mode $=9 \quad$ (This number occurs most often)

## Measure of Spread

$$
\text { Range }=16-5=11
$$

(A bigger range means the data is more spread out)

## Questions

1) Find the mean, mode, median and range of:

$$
\begin{array}{ll}
\text { a) } 3,12,4,6,8,5,4 & \text { b) } 12,1,10,1,9,3,4,9,7,9
\end{array}
$$

2) For the table:
a) Draw a pie chart to show the data.
b) Draw a bar chart to show the data.
c) Work out the mean of the data.

| Age | Frequency |
| :---: | :---: |
| 11 | 17 |
| 12 | 11 |
| 13 | 8 |

M841, M940, M934,
M328, M440, M127,
M287, M899, M460,

## TYPES OF DATA AND GRAPHS

## Key Concepts

Qualitative data: data collected that is described in words not numbers. e.g. race, hair colour, ethnicity.

Quantitative data: this is the collection of numerical data that is either discrete or continuous.
Discrete data: numerical data that is categorised into a finite number of classifications.
e.g. number of siblings in a family, shoe size,

Continuous data: numerical data that can take any value. This data is usually measured on a large number scale.
e.g. height, weight, time, capacity.


## sparx <br> U363 U557 <br> U506 U508 U983 U814

## Key Words Data <br> Discrete <br> Continuous <br> Qualitative <br> Quantitative Graph

What types of data is each of the following?

1) Eye colour
2) Length of a car (to the nearest cm )
3) Time it takes to run 100 m
4) Number of pets a person owns
5) Number of goals scored in a match

## PIE CHARTS AND SCATTER-GRAPHS



## BAR CHARTS AND PICTOGRAMS



## AVERAGES FROM A LIST AND REVERSE MEAN

## Key Concepts

There are three types of average that we use to analyse and compare data. We can calculate averages from a discrete data set.

Mode The most common value that appears in the list.

Median Once ordered, the middle value.

Mean

$$
\frac{\text { Total of all data }}{\text { Number of pieces of data }}
$$

The range is used to analyse the spread of a data set or how consistent the data is.

## Range

largest data value - smallest data value
Key Words
Discrete
Data
Mean
Mode
Median
Range
Spread

## Examples

Here is a discrete data set, calculate the mean, mode, median and range for this data.


Range: $9-2=7$

## Reverse mean

A hockey team scored the following number of goals in 6 games:

$$
\begin{array}{llllll}
2 & 3 & 4 & 1 & 0
\end{array}
$$

$$
1
$$

The mean of the goals scored in seven games was 2 . How many goals were scored in the seventh game?

$$
\frac{2+3+4+1+0+1+x}{7}=2 \longrightarrow \frac{11+x}{7}=2 \longrightarrow x=3
$$

1) Calculate the mean, mode, median and range for the following $\begin{array}{llllll}\text { list } \begin{array}{l}\text { of data: } \\ 5\end{array} & 8 & 4 & 2 & 8\end{array}$
2) The points scored in a test by 5 students are $32,38,21,25,29$. Another students test score is included. If the mean of these 6 scores is now 27 , what did the $6^{\text {th }}$ student score?

## AVERAGES FROM A TABLE

## Key Concepts

## Modal class (mode)

Group with the highest frequency.

## Median group

The median lies in the group which holds the $\frac{\text { total frequency }+1}{2}$ position. Once identified, use the cumulative frequency to identify which group the median belongs from the table.

## Estimate the mean

For grouped data, the mean can only be an estimate as we do not know the exact values in each group. To estimate, we use the midpoints of each group and to calculate the mean we find $\frac{\text { total } f x}{\text { total } f}$

## Examples

| Length <br> $(L \mathrm{~cm})$ | Frequency <br> $(\boldsymbol{f})$ | Midpoint <br> $(\boldsymbol{x})$ | $\boldsymbol{f} \boldsymbol{x}$ |
| :---: | :---: | :---: | :---: |
| $0<L \leq 10$ | 10 | 5 | $10 \times 5=50$ |
| $10<L \leq 20$ | 15 | 15 | $15 \times 15=225$ |
| $20<L \leq 30$ | 23 | 25 | $23 \times 25=575$ |
| $30<L \leq 40$ | 7 | 35 | $7 \times 35=245$ |
| Total | 55 |  | 1095 |

a) Estimate the mean of this data. step 1: calculate the total frequency step 2: find the midpoint of each group step 3: calculate $f \times x$ step 4: calculate the mean shown below

$$
\frac{\text { Total } f x}{\text { Total } f}=\frac{1095}{55}=19.9 \mathrm{~cm}
$$

b) Identify the modal class from this data set. " the group that has the highest frequency " Modal class is $20<x \leq 30$
c) Identify the group in which the median would lie. Median $=\frac{\text { Total frequency }+1}{2}=\frac{56}{2}=28$ th value " add the frequency column until you reach the $\mathbf{2 8}^{\text {th }}$ value" Median is the in group $20<x \leq 30$

## SOPMN

Key Words
Midpoint
Mean
Median
Modal


From the data:
a) Identify the modal class.
b) Identify the group which holds the median.
c) Estimate the mean.

## LISTING OUTCOMES AND SAMPLE SPACE

## Key Concepts

When there are a number of different possible outcomes in a situation we need a logical and systematic way in which to view them all.

We can be asked to list all possible outcomes e.g. choices from a menu, order in which people finish a race.

We can also use a sample space diagram. This records the possible outcomes of two different events happening

## Examples

Two dice are thrown and the possible outcomes are shown in

| Starter | Main |
| :---: | :---: |
| Fishcake <br> Melon | Lasagne <br> Beef <br> Salmon |

List all of the combinations possible when one starter and one main are chosen.

| $F, L$ | $M, L$ |
| :--- | :--- |
| $F, B$ | $M, B$ |
| $F, S$ | $M, S$ |

Note: You can write the initials of each option in a test. You do not need to write out the full word.
the sample space diagram below:

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $(1,1)$ | $(1,2)$ | $(1,3)$ | $(1,4)$ | $(1,5)$ | $(1,6)$ |
| $\mathbf{2}$ | $(2,1)$ | $(2,2)$ | $(2,3)$ | $(2,4)$ | $(2,5)$ | $(2,6)$ |
| $\mathbf{3}$ | $(3,1)$ | $(3,2)$ | $(3,3)$ | $(3,4)$ | $(3,5)$ | $(3,6)$ |
| $\mathbf{4}$ | $(4,1)$ | $(4,2)$ | $(4,3)$ | $(4,4)$ | $(4,5)$ | $(4,6)$ |
| $\mathbf{5}$ | $(5,1)$ | $(5,2)$ | $(5,3)$ | $(5,4)$ | $(5,5)$ | $(5,6)$ |
| $\mathbf{6}$ | $(6,1)$ | $(6,2)$ | $(6,3)$ | $(6,4)$ | $(6,5)$ | $(6,6)$ |

1) What is the probability that 2 numbers which are the same are rolled?

$$
\frac{6}{36}=\frac{\text { outcomes where numbers are the same }}{\text { total number of outcomes }}
$$

2) What is the probability that two even numbers are rolled?
$\frac{9}{36}=\underline{\text { outcomes where numbers are both even }}$ $\overline{36}=\frac{\text { total number of outcomes }}{}$

## sparx

U104
U296

## Key Words List

## Outcome

Sample
space
Probability

1) Abe, Ben and Carl have a race. List all of the options for the order that the boys can end the race.

|  |  | Spinner |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\div \bar{\circ} \mathrm{O}$ |  | Red | Green | Blue |
|  | Heads | H,R | H,G | H,B |
|  | Tails | T,R | T,G | T,B |

2a) What is the probability that a head is landed on? b) What is the probability that a head and a green are landed on?

## VENN DIAGRAMS



## STATISTICAL DIAGRAMS

## Key Concepts

A frequency polygon is a line graph which connects the midpoints of grouped data.
A pie chart represents data into proportional sections.

A scatter-graph shows the relationship between two variables. Correlation is used to describe the relationships.




Examples

| Answer | Frequency | Angle |
| :---: | :---: | :---: |
| Yes | 60 | 240 |
| No | 10 | 40 |
| Maybe | 20 | 80 |
| Total | 90 | 360 |


a) What type of correlation is shown? Positive correlation
b) Another student spent 6 hours revising for the test. Find an estimate of their test score.

$$
\text { Draw a line of best fit and read from it - } 68 \%
$$

c) Explain why it might not be sensible to use the scatter graph to estimate the score for a student that spent 15 hours revising. It is out of the data range.
sparx
U840, U508,
U172, U854, U277, U128

2) Draw a pie ch
using this data.


3a) What type of correlation is shown?
b) The distance from London of a house is 22 km . What is an estimate of the rent it will cost?

## CUMULATIVE FREQUENCY AND BOX PLOTS



## HISTOGRAMS

## Key Concepts

A Histogram is a graphical
representation of data consisting of rectangles whose area is proportional to the frequency of a variable and whose width is equal to the group width.


A group of people are weighed and their results recorded. Below is their data. A histogram is used to represent this data.

| Weight | Frequency | Frequency <br> density |
| :---: | :---: | :---: |
| $50<w \leq 65$ | 30 | $30 \div 15=2$ |
| $65<w \leq 70$ | 30 | $30 \div 5=6$ |
| $70<w \leq 75$ | 40 | $40 \div 5=8$ |
| $75<w \leq 85$ | 40 | $40 \div 10=4$ |
| $85<w \leq 100$ | 15 | $15 \div 15=1$ |

Example

sparx
U983
U814


Calculate the frequency density for this table of information.

On a separate set of axes, draw your histogram.

## TWO WAY TABLES AND PROBABILITY TABLES

## Key Concepts

Two way tables are used to tabulate a number of pieces of information.

Probabilities can be formulated easily from two way tables.

Probabilities can be written as a fraction, decimal or a percentage however we often work with fractions. You do not need to simplify your fractions in probabilities.

Estimating the number of times an event will occur

Probability $\times$ no. of trials

U981

## Examples

There are only red counters, blue counters, white counters and black counters in a bag.

| Colour | Red | Blue | Black | White |
| :---: | :---: | :---: | :---: | :---: |
| No. of <br> counters | 9 | $3 x$ | $x-5$ | $2 x$ |

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$
\begin{aligned}
9+3 x+x-5+2 x & =100 \\
6 x+4 & =100 \\
x & =16
\end{aligned}
$$

Number of black counters $=16-5$
= 11
Probability of choosing black $=\frac{11}{100}$

80 children went on a school trip. They went to London or to York.
23 boys and 19 girls went to London. 14 boys went to York.

|  | London | York | Total |
| :---: | :---: | :---: | :---: |
| Girls | 19 | $\mathbf{2 4}$ | $\mathbf{4 3}$ |
| Boys | 23 | 14 | $\mathbf{3 7}$ |
| Total | $\mathbf{4 2}$ | $\mathbf{3 8}$ | 80 |

What is the probability that a person is chosen that went to London? $\frac{42}{80}$
If a girl is chosen, what is the probability that she went to York? $\frac{24}{38}$

Key Words Two way table Probability Fraction Outcomes Frequency

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :---: | :---: | :---: | :---: |
| Prob | 0.37 | $2 x$ | $x$ |

1a) Calculate the probability of choosing a 2 or a 3.
b) Estimate the number of times a 2 will be chosen
if the experiment is repeated 300 times.

2a) Complete the two way table:

|  | Year Group |  |  | Total |
| :--- | :---: | :---: | :---: | :---: |
|  | 9 | 10 | 11 |  |
| Boys |  |  | 125 | 407 |
| Girls |  | 123 |  |  |
| Total | 303 | 256 |  | 831 |

b) What is the probability that a Y 10 is chosen, given that they are a girl .

## FOUR OPERATIONS WITH FRACTIONS

## Key Concept

Mixed numbers
These are made up of a whole number and a fraction.
$4 \frac{3}{5}$
$=\frac{4 \times 5+3}{5}$
$=\frac{23}{5}$
sparx
M671, M939, M601,
M835, M931, M157,

Key Words
Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).


## Tip

- A larger denominator does not mean a larger fraction.
- To find equivalent
fractions multiply/divide the numerator and denominator by the same
number.


## Examples

$$
\frac{3}{5}+\frac{2}{7}
$$

$$
\frac{3}{5}-\frac{2}{7}
$$

Make the denominators the same $\frac{3}{5}+\frac{2}{7}$
same


$$
\frac{2}{\frac{3}{5}}-\frac{2}{7}
$$



Just multiply the tops and bottoms

$$
=\frac{3 \times 2}{5 \times 7}=\frac{6}{35}
$$

Flip the second fraction and change
to a times

$$
\frac{3}{5} \times \frac{7}{2}=\frac{21}{10}
$$

3) 4 2) 5 (3) $\frac{7}{7}$
4) $\frac{7}{9}-\frac{2}{5}$
5) $\frac{3}{7} \times \frac{4}{9}$

$$
\text { 5) } \frac{3}{11} \div \frac{14}{22}
$$

## INTEGERS, ROUNDING AND PLACE VALUE



## DECIMALS



## CUMULATIVE FREQUENCY AND BOX PLOTS



## RATIONALISE THE DENOMINATOR

## Key Concepts

A surd can be written within a fraction.
However, we do not want an irrational number on the denominator of a fraction therefore we must rationalise it.

To rationalise a surd we can multiply it by itself.

Rationalise $\frac{1}{\sqrt{5}}$

## Examples

$$
\begin{gathered}
\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\
=\frac{\sqrt{5}}{5}
\end{gathered}
$$

Rationalise $\frac{5}{2 \sqrt{3}}$

$$
\begin{aligned}
& \frac{5}{2 \sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
= & \frac{5 \sqrt{3}}{2 \times 3}=\frac{5 \sqrt{3}}{6}
\end{aligned}
$$

## Change the sign

$$
\begin{aligned}
& \text { Rationalise } \frac{2+\sqrt{3}}{3-\sqrt{5}} \\
& \frac{2+\sqrt{3}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} \\
& =\frac{(2+\sqrt{3})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} \\
& =\frac{6+3 \sqrt{3}+2 \sqrt{5}+\sqrt{15}}{9-3 \sqrt{5}+3 \sqrt{5}-5} \\
& =\frac{6+3 \sqrt{3}+2 \sqrt{5}+\sqrt{15}}{4}
\end{aligned}
$$

1) Rationalise $\frac{1}{\sqrt{7}}$
2) Rationalise $\frac{4+\sqrt{5}}{\sqrt{2}}$
3) Rationalise $\frac{3}{2 \sqrt{5}}$
4) Rationalise $\frac{2-\sqrt{2}}{1+\sqrt{5}}$

## Year 10 Higher SURDS



