

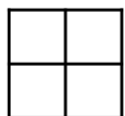
# POWERS AND ROOTS

## Key Concept

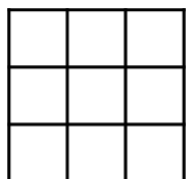
Square numbers



$$1^2 \\ 1 \times 1 = 1$$



$$2^2 \\ 2 \times 2 = 4$$



$$3^2 \\ 3 \times 3 = 9$$

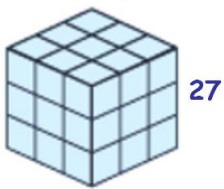
Cube numbers



$$1^3 \\ 1 \times 1 \times 1$$



$$2^3 \\ 2 \times 2 \times 2$$



$$3^3 \\ 3 \times 3 \times 3$$

## Key Words

**Square:** A square number is the result of multiplying a number by itself.

**Cube:** A cube number is the result of multiplying a number by itself twice.

**Root:** A root is the reverse of a power.

**Prime number:** A prime is a number that has only two factors which are 1 and itself.

**Reciprocal:** This is found by doing 1 divided by the number.

**Factor:** A number that fits into another number exactly.

## Tip

A number with an odd amount of factors must be a square number.

## Examples

What is  $2^4$  ?

$$2 \times 2 \times 2 \times 2 = 16$$

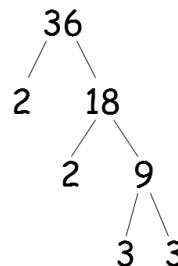
What is  $\sqrt{64}$  ?

$$8^2 = 64, \text{ so } \sqrt{64} = \pm 8$$

What is the reciprocal of 5?

$$\frac{1}{5}$$

Write 36 as a product of prime factors



$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

Product means 'multiply'

sparx

M823, M322  
M108

## Questions

- a)  $2^5$    b)  $3^3$    c)  $1^{17}$    d)  $\sqrt{81}$    e)  $\sqrt{16}$    f)  $\sqrt[3]{64}$
- Find the reciprocal of:   a) 4   b)  $\frac{1}{3}$    c) 0.25
- Write 72 as a product of primes.

ANSWERS: 1) a) 32   b) 27   c) 1   d)  $\pm 9$    e)  $\pm 4$    f) 4  
2) a)  $\frac{1}{4}$    b) 3   c) 4  
3)  $2^3 \times 3^2$

# INDICES AND ROOTS

## Key Concepts

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{-m} = \frac{1}{a^m}$$

## Examples

Simplify each of the following:

$$1) a^6 \times a^4 = a^{6+4} \\ = a^{10}$$

$$4) (3a^4)^3 = 3^3 a^{4 \times 3} \\ = 27a^{12}$$

$$6) a^{\frac{1}{2}} = \sqrt{a}$$

$$2) a^6 \div a^4 = a^{6-4} \\ = a^2$$

$$5) \frac{5^2 \times 5^6}{5^4} = \frac{5^8}{5^4} \\ = 5^{8-4}$$

$$7) 9^{\frac{1}{2}} = \sqrt{9} \\ = 3 \text{ or } -3$$

$$3) (a^6)^4 = a^{6 \times 4} \\ = a^{24}$$

$$= 5^4$$

$$8) 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

**sparx**  
M135, M608,  
M105, M608,  
M150

## Key Words

Powers  
Roots  
Indices  
Reciprocal

Simplify:

$$1) a^3 \times a^2 \quad 2) b^4 \times b \quad 3) d^{-5} \times d^{-1} \quad 4) m^6 \div m^2 \quad 5) n^4 \div n^4$$

$$6) \frac{8^4 \times 8^5}{8^6} \quad 7) \frac{4^9 \times 4}{4^3} \quad 8) (3^2)^5 \quad 9) 81^{\frac{1}{2}} \quad 10) 5^{-2}$$

# ALGEBRAIC EXPRESSIONS

## Key Concepts

When collecting like terms involving addition or subtraction, add/subtract the numbers in front of the letters.

If the like terms are multiplied, multiply the numbers in front of the letters and put the letters next to each other.

If the like terms are divided, divide the numbers in front of the letters.

## Examples

Simplify the following expressions:

$$1) 4p + 6t + p - 2t = 5p + 4t$$

$$2) 3 + 2t + p - t + 2 = 5 + t + p$$

$$3) f + 3g - 4f = 3g - 3f$$

$$4) f^2 + 4f^2 - 2f^2 = 3f^2$$

$$5) 6a \times 3b \times 2c = 36abc$$

$$6) \frac{9b}{3} = 3b$$

**sparx**

M813, M795,  
M531, M949

## Key Words

Simplify  
Term  
Collect

## Questions

Simplify:

$$1) 7p + 3q + p - 3q$$

$$3p - 2t + 7$$

$$3) m - 8g - 5m$$

$$+ 2b^2$$

$$5) 2a \times 5b \times 4c$$

$$3n \times 2m$$

$$2) 5 + 4t +$$

$$4) b^2 - 7b^2$$

$$6) 8m \times$$

# EXPAND AND SIMPLIFY BRACKETS

## Key Concepts

### Expanding brackets

Multiply the number outside the brackets with EVERY term inside the brackets

### Factoring expressions

Take the highest common factor outside the bracket.

## Examples

Expand and simplify where appropriate

1)  $7(3 + a) = 21 + 7a$

2)  $2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a$   
 $= 16 + 5a$

3) Factorise  $9x + 18 = 9(x + 2)$

4) Factorise  $6e^2 - 3e = 3e(2e - 1)$

## Questions

### 1) Expand and simplify

- (a)  $3(2 - 7f)$       (b)  $5(m - 2) + 6$       (c)  $3(4 + t) + 2(5 + t)$

### 2) Factorise

- (a)  $6m + 12t$       (b)  $9t - 3p$       (c)  $4d^2 - 2d$

**sparx**

M237, M100, M208,  
M608, M150

## Key Words

- Expand
- Factorise
- Simplify

# SIMPLIFYING & MANIPULATING ALGEBRA

## Key Concept

**Formula**

$$v = u + at$$

**Expression**

$$f^2 + f^2 + f^2$$

**Equation**

$$34 = 12 + 6t$$

**Identity**

$$c \times c = c^2$$

# sparx

U330,U534,  
M635,M690

## Key Words

**Formula:** A rule written using symbols that describe a relationship between different quantities.

**Expression:** Shows a mathematical relationship whereby there is no solution.

**Equation:** A mathematical statement that shows that two expressions are equal.

**Identity:** A relation which is true. No matter what values are chosen.

## Tip

When expanding brackets be careful with negatives.

## Examples

Simplify:

$$4a + 3b - a + 2b = 3a + 5b$$

Expand and simplify:

$$9a - 2(3a - 4) = 9a - 6a + 8 = 3a + 8$$

Factorise:

$$9x^2 + 6x$$

Factorising is the opposite of expanding brackets

3x is common to both terms

$$3x(3x + 2)$$

Expand and simplify:

$$2(4a + 2b) - 2(a + 3b)$$

$$8a + 4b - 2a - 6b = 6a - 2b$$

## Questions

- 1)  $5x + 3y - 2x + 4y$     2)  $2p - 6q + 2q + 4p$     3)  $12b - 3(2b + 5)$   
 4) Factorise a)  $4x + 10$     b)  $8a^2 - 10a$

ANSWERS: 1)  $3x + 7y$     2)  $6p - 4q$     3)  $6b - 15$   
 4) a)  $2(2x + 5)$     b)  $2a(4a - 5)$

# SOLVING EQUATIONS

## Key Concept

### Inverse Operations

Operation	Inverse
+	-
-	+
x	÷
÷	x
$x^2$	$\sqrt{x}$

## Key Words

**Unknown:** A letter which represents a number we do not know the value of.

**Terms:** The numbers and letters in the expression or equation.

**Inverse:** The operation which will do the opposite.

## Examples

$x + 9 = 16$ $-9 \quad -9$ $x = 7$	$x - 12 = 20$ $+12 \quad +12$ $x = 32$	$\frac{x}{3} = 5$ $\times 3 \quad \times 3$ $x = 15$	$2x + 5 = 14$ $-5 \quad -5$ $2x = 9$ $\div 2 \quad \div 2$ $x = 4.5$
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$\frac{x}{4} - 2 = 4$ $+2 \quad +2$ $\frac{x}{4} = 6$ $\times 4 \quad \times 4$ $x = 24$	$2(3x + 5) = -14$ <b>expand</b> $6x + 10 = -14$ $-10 \quad -10$ $6x = -24$ $\div 6 \quad \div 6$ $x = -4$	$2x + 7 = 5x + 1$ $-2x$ <b>(smallest x term)</b> $+7 = 3x + 1$ $-1 \quad -1$ $6 = 3x$ $\div 3 \quad \div 3$ $2 = x$
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# sparx

M707, M509,  
M554

## Tip

Answers can be:

- Integers
- Decimals
- Fractions
- negatives

## Questions

- $x + 8 = 19$
- $y - 25 = 15$
- $2y = 82$
- $\frac{t}{4} = 7$
- $\frac{p}{2} - 6 = 2$
- $3(2x - 3) = 15$
- $4x - 8 = 2x + 1$

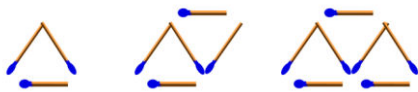
ANSWERS: 1)  $x = 11$ , 2)  $y = 40$ , 3)  $y = 41$ , 4)  $t = 28$ , 5)  $p = 16$ , 6)  $x = 4$ , 7)  $x = 4.5$

# SEQUENCES

## Key Concept

### Types of Sequence

Sequence as pictures:



Linear sequence:

4, 7, 10, 13, 16, ...



Fibonacci sequence:

(add the previous two terms)

1, 1, 2, 3, 5, 8, ...

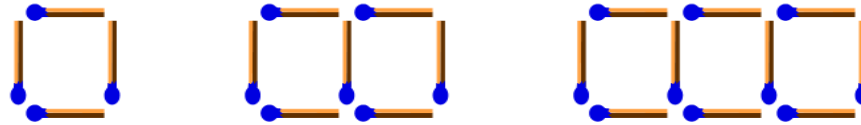
## Key Words

**Sequence:** A list which is in a particular order following a pattern.

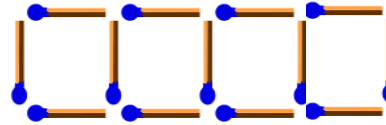
**Term:** Each particular part of a sequence.

**Linear sequence:** A sequence which is formed by adding or subtracting the same amount each time.

## Examples



Next pattern is:



Sequence = 4, 7, 10, 13, ...

Term to term rule = + 3

Nth term

$$4, 7, 10, 13, 16, \dots = 3n + 1$$

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M241, M381,  
M991

## Tip

If a sequence is decreasing, the 'n' term will be negative.

Eg, 15, 11, 7, 3, ...

Nth term =  $-4n + 19$

## Questions

1) Find the next two terms and the term to term rule

a) 9, 13, 17, 21, ...    b) 7, 12, 17, 22, ...    c) 9, 7, 5, 3, ...    d) 3, 4, 7, 11, 18

2) Find the nth term    a) 7, 9, 11, 13, ...    b) 8, 13, 18, 23, ...

c) 15, 12, 9, 6, ...    d) 1, -3, -7, -11, ...

ANSWERS: 1) a) 25, 29 Rule = +4    b) 27, 32, Rule = +5    c) 1, -1, Rule = -2    d) 29, 47, Rule = add previous 2 numbers    2) a)  $2n + 5$     b)  $5n + 3$     c)  $-3n + 18$     d)  $-4n + 5$