

# FRACTIONS & PERCENTAGES AS OPERATORS

## Key Concept

### Multipliers

Find 15%	$\times 0.15$
Increase by 15%	$\times 1.15$
Decrease by 15%	$\times 0.85$

For **reverse percentage** problems you can divide by the multiplier to find the original amount.

## Key Words

**Percentage:** Is a proportion that shows a number as parts per hundred.

**Fraction:** A fraction is made up of a numerator (top) and a denominator (bottom).

**Multiplier:** A quantity by which a given number is to be multiplied.

## Examples

### Non-Calculator

$$\frac{3}{4} \text{ of } 32 = 32 \div 4 \times 3 = 24$$

$$\begin{array}{l} 16\% \text{ of } 240 \\ 10\% = 24 \\ 5\% = 12 \\ 1\% = 2.4 \end{array} \left. \vphantom{\begin{array}{l} 16\% \text{ of } 240 \\ 10\% = 24 \\ 5\% = 12 \\ 1\% = 2.4 \end{array}} \right\} \begin{array}{l} = 24 + 12 + 2.4 \\ = 38.4 \end{array}$$

### Calculator

Find **32%** of 54.60 = **0.32**  $\times$  54.60 = 17.472

Increase 45 by **12%** = 45  $\times$  **1.12** = 50.4

**sparx**

M157,U475

M958,M264,U88

M437

## Tip

There is a % function on your calculator.

To find 25% of 14 on a calculator:

**2, 5, SHIFT, (,  $\times$ , 1, 4, =**

## Questions

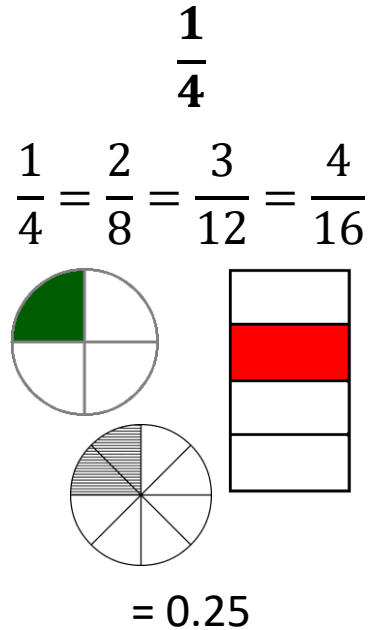
1) Find these fractions of amounts:

a)  $\frac{1}{3}$  of 15    a)  $\frac{1}{5}$  of 65    a)  $\frac{2}{7}$  of 14    a)  $\frac{4}{9}$  of 45

2) a) 35% of 140    b) 21% of 360    c) Increase 60 by 15%

# UNDERSTANDING FRACTIONS

## Key Concept



## Key Words

**Fraction:** A fraction is made up of a numerator (top) and a denominator (bottom).

**Equivalence:** Two fractions are equivalent if one is a multiple of the other.

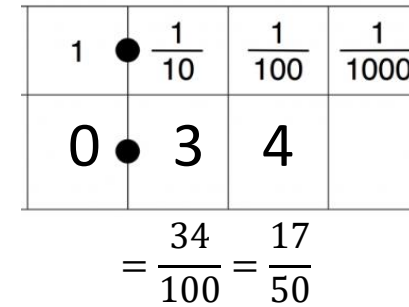
**Simplify:** Cancel a fraction down to give the smallest numbers possible.

## Examples

Simplify  $\frac{3}{24}$   
a fraction

$$\frac{3}{24} \xrightarrow{\div 3} \frac{1}{8}$$

Write 0.34 as



Write  $\frac{6}{7}$  as a decimal

$$\frac{6}{7} \rightarrow 7 \overline{) 6.000000}$$

6 4 5 1 3

What is  $\frac{3}{5}$  of 45

$$45 \div 5 \times 3 = 27$$

# sparx

M158, M410,  
M601, M835,  
M157, M197

## Tip

- A larger denominator does not mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

## Questions

- 1) Simplify a)  $\frac{42}{96}$  b)  $\frac{64}{120}$
- 2) Write as a decimal a)  $\frac{2}{7}$  b)  $\frac{3}{8}$
- 3) Write as a fraction a) 0.48 b) 0.166 c) 0.308
- 4) a)  $\frac{3}{5}$  of 35 b)  $\frac{2}{9}$  of 45 c)  $\frac{5}{11}$  of 121

ANSWERS : 1) a)  $\frac{16}{8}$  b)  $\frac{16}{7}$  c)  $\frac{12}{25}$  2) a) 0.2857... b) 0.375  
3) a)  $\frac{12}{77}$  b)  $\frac{500}{83}$  c)  $\frac{250}{77}$  4) a) 21 b) 10 c) 55

# FOUR OPERATIONS WITH FRACTIONS

## Key Concept

### Mixed numbers

These are made up of a whole number and a fraction.



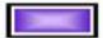

$$4\frac{3}{5}$$

$$= \frac{4 \times 5 + 3}{5}$$

$$= \frac{23}{5}$$

## Key Words

**Fraction:** A fraction is made up of a numerator (top) and a denominator (bottom).

 <p>Add Sum Total All together Plus In all</p>	 <p>Multiply Product Times Twice Total Multiplied by</p>
 <p>Subtract Remain Difference Less than Fewer How many more Minus</p>	 <p>Divide Quotient Goes into Split Equally Each</p>

## Examples

$$+\quad \frac{3}{5} + \frac{2}{7}$$

Make the denominators the same

$$\begin{array}{r} \frac{3}{5} + \frac{2}{7} \\ \times 7 \quad \times 5 \\ \hline \frac{21}{35} + \frac{10}{35} = \frac{31}{35} \end{array}$$

$$-\quad \frac{3}{5} - \frac{2}{7}$$

$$\begin{array}{r} \frac{3}{5} - \frac{2}{7} \\ \times 7 \quad \times 5 \\ \hline \frac{21}{35} - \frac{10}{35} = \frac{11}{35} \end{array}$$



$$\times \quad \frac{3}{5} \times \frac{2}{7}$$

Just multiply the tops and bottoms

$$= \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$$

$$\div \quad \frac{3}{5} \div \frac{2}{7}$$

Flip the second fraction and change to a times

$$\frac{3}{5} \times \frac{7}{2} = \frac{21}{10}$$

# sparx

M671, M939, M601,  
M835, M931, M157,  
M197, M110

## Tip

- A larger denominator **does not** mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

## Questions

1)  $\frac{3}{5} + \frac{4}{15}$     2)  $\frac{2}{7} + \frac{5}{8}$     3)  $\frac{7}{9} - \frac{2}{5}$     4)  $\frac{3}{7} \times \frac{4}{9}$     5)  $\frac{3}{11} \div \frac{14}{22}$

5)  $\frac{7}{3}$

4)  $\frac{21}{4}$

3)  $\frac{17}{45}$

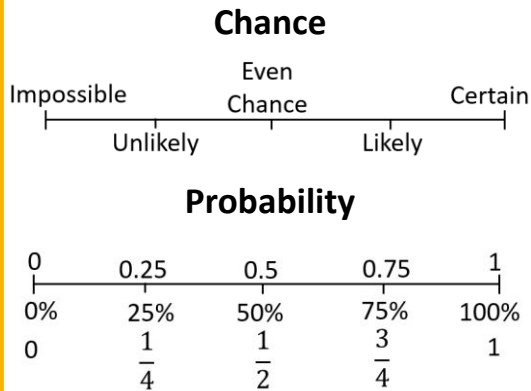
2)  $\frac{51}{8}$

1)  $\frac{13}{15}$

ANSWERS:

# INTRODUCING PROBABILITY

## Key Concept



Probabilities can be written as:

- Fractions
- Decimals
- Percentages

# sparx

Clip Numbers

M655, M941,

M938

## Key Words

**Probability:** The chance of something happening as a numerical value.

**Impossible:** The outcome cannot happen.

**Certain:** The outcome will definitely happen.

**Even chance:** There are two different outcomes each with the same chance of happening.

**Expectation:** The amount of times you expect an outcome to happen based on probability.

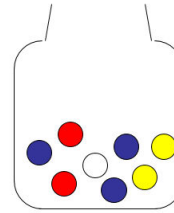
## Tip

Probabilities always add up to 1.

## Formula

*Expectation*  
= *Probability* × *no. of trials*

## Examples

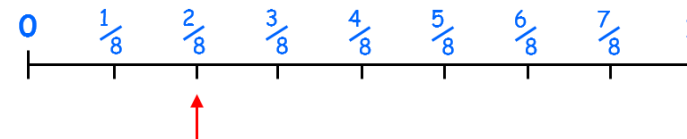


1) What is the probability that a bead chosen will be **yellow**.

Show the answer on a number line.

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{Yellow}) = \frac{2}{8} = \frac{1}{4}$$



2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} \text{ of } 40 = 10$$

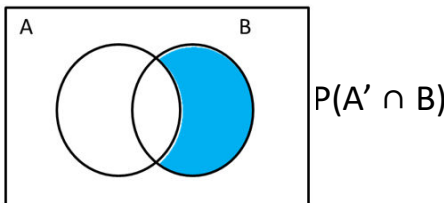
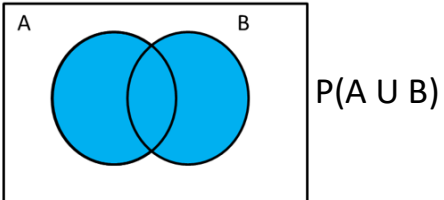
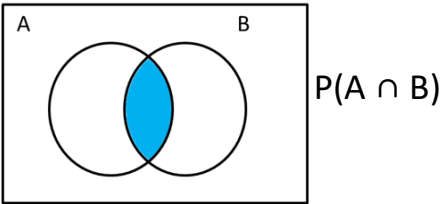
## Questions

In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left. Find: a) P(Red) b) P(Yellow) c) P(Red or purple) d) P(Green)

ANSWERS: 1) a)  $\frac{30}{12}$  b)  $\frac{5}{2}$  c)  $\frac{30}{9}$  d)  $\frac{10}{3}$  2) a)  $\frac{15}{1}$  b)  $\frac{30}{1}$  c)  $\frac{10}{1}$  d)  $\frac{2}{1}$

# FURTHER PROBABILITY

## Key Concept



## Key Words

**Probability:** The chance of something happening as a numerical value.

**Impossible:** The outcome cannot happen.

**Certain:** The outcome will definitely happen.

**Even chance:** There are two different outcomes each with the same chance of happening.

**Mutually Exclusive:** Two events that cannot both occur at the same time.

## Formula

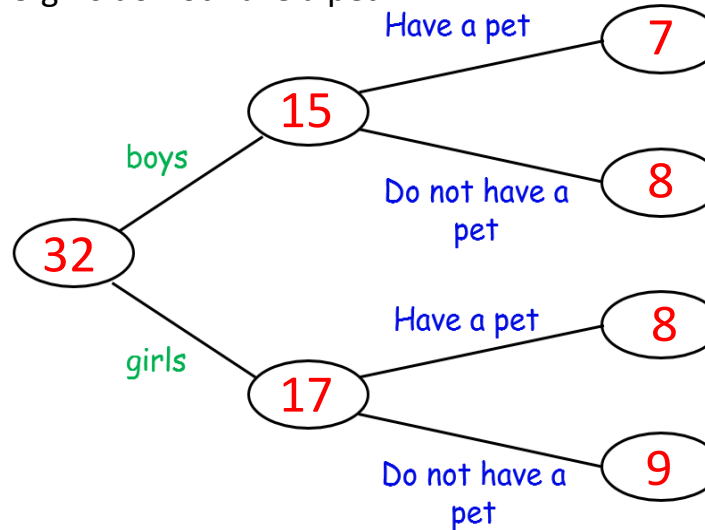
$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

or (non ME)  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

## Examples

In Hannah's class there are 32 students.  
15 of these students are boys.  
7 of the boys have a pet.  
9 girls do not have a pet.



$$P(\text{boy}) = \frac{15}{32}$$

$$P(\text{Girl with pet}) = \frac{8}{32}$$

**sparx**

M718, M419

M829, M460

## Questions

- 1) Draw a two-way table for the question above.
- 2) Find the probability that a pupil chosen is a boy with no pets.
- 3) A girl is chosen, what is the probability she has a pet?

3)  $\frac{17}{32}$

2)  $\frac{32}{8}$

ANSWERS:

# FACTORS, MULTIPLES AND PRIMES

## Key Concept

### Factors:

Find these in pairs

**12**  
1, 12  
2, 6  
3, 4

### Multiples:

Start with the number itself  
**7** – 7, 14, 21, 28, ...

## Key Words

**Factor:** The numbers which fit into a number exactly.

**Multiple:** The numbers in the times table.

**Prime:** Numbers which have only two factors which are 1 and itself.

**Highest Common Factor:** The highest factor which is common for both numbers.

**Lowest Common Multiple:** The smallest multiple which is common to both numbers.

## Examples

### Lowest Common Multiple (LCM)

Q - Find the LCM of 6 and 7:

6 – 6, 12, 18, 24, 30, 36, **42**, 48, 54, 60, ...

7 – 7, 14, 21, 28, 35, **42**, 49, 56, ...

LCM = 42

### Highest Common Factor (HCF)

Q – Find the HCF of 18 and 24

18 – 1, 2, 3, **6**, 9, 18

24 – 1, 2, 3, 4, **6**, 8, 12, 24

HCF = 6

**sparx**

M462

M823

M322

## Tip

There is only one even prime number which is the number 2. This can be used to help solve lots of problems.

## Questions

- 1) List the first 5 multiples of: a) 7 b) 12 c) 50
- 2) List the factors of: a) 12 b) 15 c) 16
- 3) a) Find the LCM of 5 and 7 b) Find the HCF of 20 and 16

ANSWERS: 1) a) 7, 14, 21, 28, 35 b) 12, 24, 36, 48, 60 c) 50, 100, 150, 200, 250  
2) a) 1, 2, 3, 4, 6, 12 b) 1, 3, 5, 15 c) 1, 2, 4, 8, 16 3) a) 35 b) 4